STATISTICAL ANALYSIS OF FLAWS IN GLAZED AND UNGLAZED CERAMIC TILES VIA THE WEIBULL DISTRIBUTION

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ABSTRACT

The three point bending strength of glazed and unglazed tiles was measured. The values of the Weibull parameters were estimated via Linear Regression (LR), Moment (M) and Maximum Likelihood (ML). The LR method showed the minimum Kolmogorov distance, indicating that it was more precise for determining the Weibull parameters. Furthermore, while the characteristic strength and the mean strengths were increased by the glazing, the Weibull modulus was reduced. The strength distributions of the glazed and the unglazed tiles were completely different, i.e. they demonstrated a bimodal and a unimodal distribution, respectively. This behaviour was attributed to new flaws originating from the glazing.

1. INTRODUCTION

Ceramics have many excellent mechanical properties such as high hardness, stiffness, elastic modulus, and wear resistance; however, the large scattering in fracture strength is their major drawback which adversely affects their development as a structural component. Indeed, the same ceramic specimens under the same loading conditions show different strength values. This wide scattering in ceramic strength values gives rise to low reliability and, when coupled with poor toughness, intensifies the probability of catastrophic failure [1-3]. The probabilistic nature of fracture in brittle ceramics is derived from the arbitrary distribution of critical microstructural flaws with a different size, shape, and orientation, which are inevitable even in high-tech processing [4].

For this reason, the mechanical performance of ceramic components should be determined statistically through an investigation of the flaw population; however, the characterization of fracture origins and the determination of flaw populations require complicated and time-consuming fractography. Since the strength data distribution maps the flaw population, the evaluation is done based on fracture strength variability [5]. The statistical investigation of fracture strength also provides a trade-off between high strength values and low variability because both are important issues in structural applications.

The best-known probability function for characterizing strength distribution in brittle materials is the Weibull distribution, which is proposed based on the weakest link hypothesis [6]. Generally, the Weibull distribution function can be used for the investigation of many phenomena, where the probability of an event in a part of an object is equal to the probability of the event in the object as a whole, like a chain that breaks when one of its weakest links fails [7]. Weibull proposed the following equation:

$$P_f = 1 - \exp\left[-\int_V \left(\frac{\sigma}{\sigma_0} \right)^m dV \right] \quad \sigma > 0$$

where, P_f is the probability of failure, σ_0 is the Weibull characteristic strength which is closely related to the average fracture strength, V is the region which is under tensile stress, and m is the Weibull modulus, which indicates the strength value scattering[8]. A lower m value leads to a wider dispersion of fracture strength, corresponding to less uniformity of flaws, while a higher m value indicates less variability in flaw size distribution and in fracture strength, so that a higher level of integrity is expected [9]. There are different methods of estimating the Weibull parameters including Linear Regression (LR), Moments and Maximum Likelihood (ML) which are discussed in [10, 11].

In this paper, we used the Weibull statistical function to analyze the fracture behaviours of two series of glazed and unglazed tiles.

2. EXPERIMENTAL PROCEDURE

The samples which were used in this experiment were randomly chosen from the products of Irana Tile Co. At least 30 identical samples were examined in each series.

The unglazed samples, referenced C1 hereafter, were fired in a fast-firing roller kiln at 1100°C with a soaking of about 5 min. The glazed series, referenced C2 hereafter, were subjected to a second glaze firing. The maximum firing temperature and total firing time were 1030°C and 40 min, respectively.

A 3-point bending test configuration was used to measure the flexural strength of the C1 and C2 groups. The nominal dimension of the C1 and C2 series was 40cm×25cm×7cm, using a span length of 38cm and crosshead speed of 0.5mm/min. The flexural strength values obtained for C1 and C2 have been tabulated in ascending order in Table 1 and Table 2 respectively.

Sample no.	Bending strength(kg/cm^2)
1	131.64
2	136.90
3	139.61
4	140.02
5	143.16
6	144.92
7	145.82
8	147.14
9	150.23
10	152.84
11	154.26
12	155.11
13	155.93
14	156.89
15	156.89
16	157.76
17	158.00
18	158.70
19	158.74
20	159.17
21	159.49
22	161.30
23	163.00
24	163.14
25	163.52
26	163.55
27	163.66
28	164.95
29	166.43
30	166.55
31	167.04
32	167.67
33	168.04

34	168.04
35	173.63

Table 1. *The measured 3-point bending strength data for unglazed tiles (C1)*

Sample no.	Bending strength (kg/cm²)
1	157.47
2	158.99
3	159.73
4	166.74
5	170.06
6	173.29
7	175.90
8	179.07
9	181.29
10	181.51
11	184.33
12	185.47
13	190.20
14	193.17
15	197.21
16	200.65
17	203.77
18	203.99
19	205.84
20	206.40
21	208.05
22	208.82
23	210.44
24	211.64
25	211.82
26	212.23
27	213.98
28	214.64
29	214.74
30	214.82
31	215.55
32	216.33
33	217.16
34	218.31
35	218.52
36	222.09

Table 2. *The measured bending strength data for glazed tiles (C2)*

3. RESULTS AND DISCUSSION

In this study, three types of estimators including Linear Regression (LR), Moment (M) and Maximum Likelihood (ML) were used to estimate the Weibull parameters including the Weibull modulus or shape parameter and scale parameter for both groups. The complete explanation of these methods can be found in [10, 11]. Different expressions were applied to define the probability estimator in the LR method. Since it has been shown that $P_f = \frac{n-0,5}{N}$ leads to the least biased estimate of the Weibull modulus, it has been used in this article [12, 13]. Table 3 shows the Weibull parameters of C1 and C2 estimated via three different estimators.

Reference	LR		M		ML	
	σ_0	m	σ_0	M	σ_0	m
C1	160.10	18.71	159.70	19.15	159.00	20.07
C2	220.75	11.75	218.56	12.22	214.03	13.57

Table 3. *The Weibull parameters of C1 and C2 estimated via three methods of LR, M, and ML.*

Table 3 reveals that different estimation methods produced different values for the Weibull parameters. The empirical cumulative distribution (failure function) and the fitted failure functions for different estimators and samples are shown in Fig. 1 and Fig. 2.

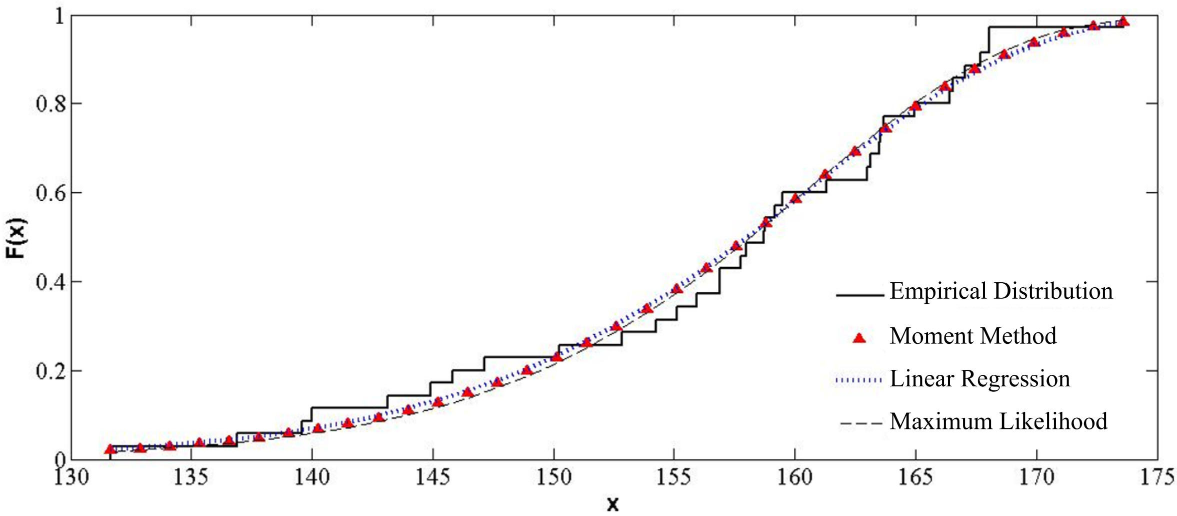

Figure 1. *Comparison of the cumulative distribution function of three estimators and empirical results for C1*

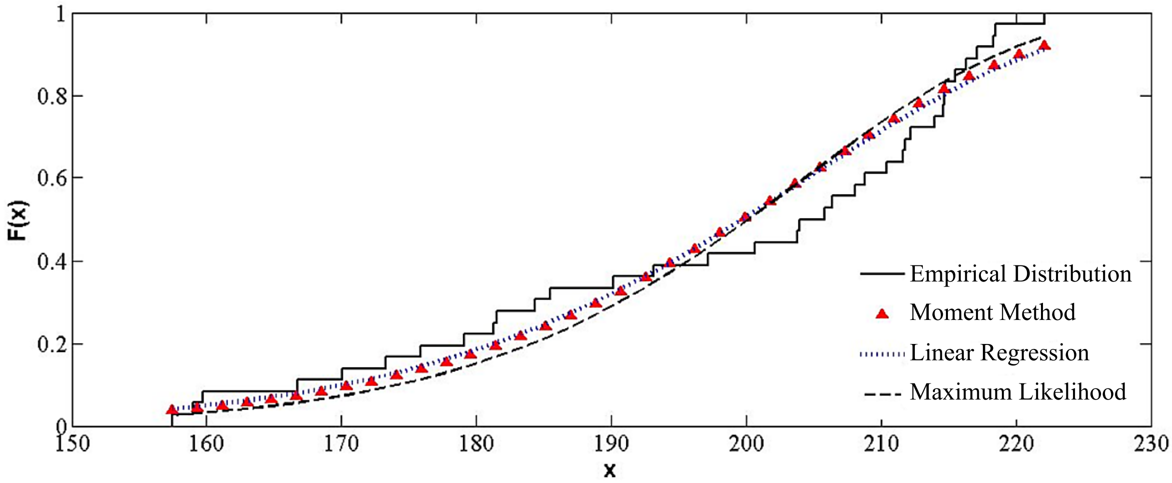

Figure 2. *Comparison of the cumulative distribution function of three estimators and empirical results for C2*

There are several criteria which are used to determine the compatibility of the fitted distribution function to the empirical data. The Kolmogorov–Smirnov test (K-S test) is one of the most useful and popular methods, and it indicates the fitness of the estimator and data[14]. The K-S distances for the three estimation methods and the samples are indicated in Table 4.

Reference	Linear Regression	Moment	Maximum Likelihood
C1	0.0841	0.0845	0.0906
C2	0.1468	0.1535	0.1586

Table 4. *The values of the Kolmogorov-Smirnov test for different estimation methods.*

Table 4 clearly shows that for both samples, the LR method gives the minimum Kolmogorov–Smirnov distance, establishing that more accuracy can be obtained by using this than by using the M and ML estimators.

Based on Tables 3 and 5, it can be observed that both the Weibull characteristic strength and the mean strengths of C2 are higher than C1. The glaze layer can cover the surface cracks of the substrate, keeping them from the external stresses and thereby improving the characteristic and mean strengths, considerably. Table 3 also presents a higher Weibull modulus (m) for C1 compared to C2, in the three different methods, which means that the C2 series suffers from a more scattered flaw size. In addition, according to Table 5, it was observed that the variance has a higher value for C2, which confirms a wider dispersion in fracture strength values of the glazed specimens.

Reference	Mean strength	Variance
C1	156.68	102.59
C2	197.34	385.39

Table 5. *Mean strength and variance values for C1 and C2*

Fig. 3 depicts the Weibull plot of the strength values for both samples. In a Weibull plot, typically, $\ln(\ln(1/(1-P_f)))$ are depicted versus $\ln(\sigma)$. As shown in Fig. 3, the strength values of C1 are appropriately fitted to a straight line; however, the data for C2 do not suitably fit to the straight line and are therefore deviated from the Weibull distribution. The results reveal that a more complex model, i.e. a bi- or multi-modal Weibull distribution is needed to describe the C2 data values, due to the severe deviation of C2 data from the Weibull unimodal distribution. As shown in Table 4, the R^2 values for C1 are higher than C2, 0.98 and 0.93 respectively, and conversely, the K-S values for C1 are lower than those of C2. These values verify the deficiency of the unimodal model to describe the distribution of the fracture strength of glazed specimens.

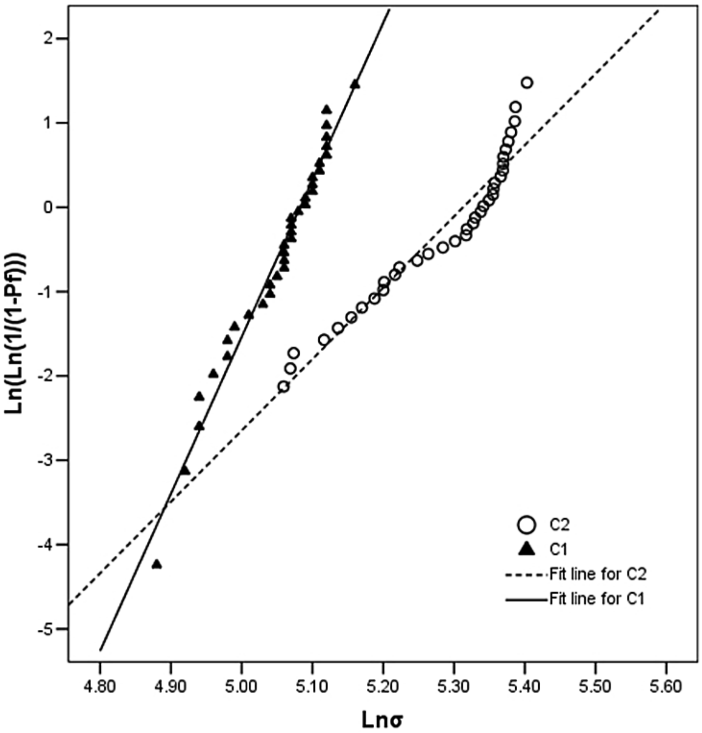

Figure 3. *Weibull plot for C1 and C2*

Consequently, the bimodal distribution of Weibull was used for further investigation. For this purpose, the strength measurements for C2 were divided into two categories, C2-1 and C2-2. The fracture strength data were classified based on the highest values for R^2 of the C2-1 and C2-2 fit lines. Fig. 4 illustrates the unimodal and bimodal strength distribution for C1 and C2, respectively.

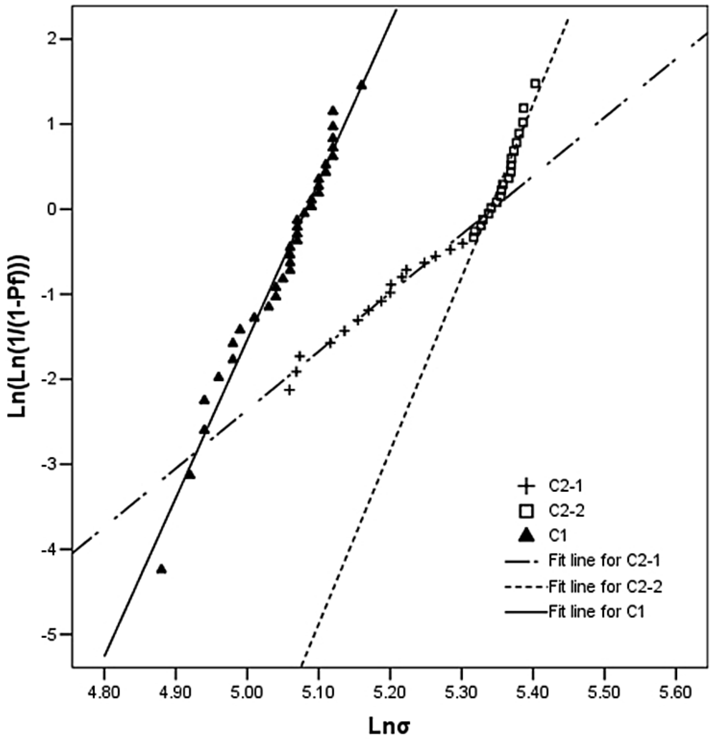

Figure 4. *Weibull plots for the C2 subgroups*

The better fitness of the bimodal distribution compared to the unimodal distribution of C2 is characterized by the variation in R^2, variance, and K-S values. As reported in Table 6, the R^2 values in C2-1 and C2-2 are higher than in C2, showing that the bimodal distribution more appropriately matches the strength data. Furthermore, a significant decrease in variance and K-S values justifies the use of the more complex model for C2.

Reference	Sample size	R^2	Variance	K-S
C2-1	19	0.98	246.31	0.1249
C2-2	17	0.95	16.849	0.0919

Table 6*. The R^2, Variance and K-S values for C2-1 and C2-2.*

It should also be noted that fitting the multimodal distribution does not significantly change the R^2 and K-S values and only complicates the model, so that it is not preferred.

As mentioned previously, based on the weakest link theory, the most serious flaw in the material determines the strength of material; consequently, the strength distribution reflects the flaw distribution. In this case, the flaw distribution in glazed and unglazed tiles is completely different. The strength values of unglazed sample are appropriately aligned along a straight line, and it is well-matched with the Weibull distribution. As a result, fracture in this group develops from a single flaw population. However, the strength data of the glazed sample follow a bimodal distribution and, therefore, fracture originates from two or more active concurrent flaw distributions. Table 7 shows the characteristic strength and Weibull modulus for the C2 subgroups.

Reference	σ_0	m
C2-1	278.62	6.61
C2-2	205.01	23.20

***Table 7**. Characteristic strength and Weibull modulus for C2-1 and C2-2*

In the glazed tile the flaws can originate from the substrate and the glaze layer. In other words, the non-uniformity of the glazed layer results in inhomogeneous flaw origins. Because the strength distribution is strongly related to flaw distribution [5], we observe a bimodal fracture strength distribution. Consequently we can conclude that glazing introduces new flaw origins and changes the flaw distribution as well as the fracture strength distribution.

Further investigation such as fractographic examination is recommended for confirmation of multiple flaw distributions and determining the flaw origins.

4. CONCLUSIONS

In the present study, we use the Weibull statistical distribution model to investigate the fracture strength of glazed and unglazed specimens. Three different methods of Linear Regression (LR), Moments and Maximum Likelihood (ML) were used to estimate the Weibull modulus and characteristic strength. The result showed that while the characteristic and the mean strengths of samples increase with glazing, the Weibull modulus decreases conversely. The strength distribution of the unglazed specimens was unimodal; however, the glazed specimens followed a bimodal distribution. Since flaw distribution and fracture strength distribution are strongly related, we have concluded that glazing produces new flaw origins.

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