# MATHEMATICAL MODELLING APPLIED TO THE DIMENSIONAL CONTROL OF CERAMIC TILES PRODUCED BY THE SINGLE FIRING AND WET MILLING ROUTE

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### 1. INTRODUCTION

The Brazilian ceramic tiles are produced according to dimensional tolerances established by national (NBR13818)<sup>[1]</sup> and international (ISO 13006)<sup>[2]</sup> standards. One of these requirements is related to dimensional uniformity of the tiles (size and form). In the case of size, the manufacturers divide the standard tolerance generally into 3 categories: small (P), medium (M) and large (G). Each category normally has 1.0 or 1.2 mm of tolerance. The challenge of the dimensional control is to produce the highest amount of tile according to the medium category size. A production of other categories does not declassify the product, but are associated to increasing of production and storage lots.

The final size, expressed by length, of a ceramic tile is a result of the dimensional variations that it suffers along the process, from conformation up to firing. These variations are associated to length of the conformation matrix (Li), linear expansion after conformation (Ex), linear drying shrinkage (Rs) and linear firing shrinkage (Rq), Equation A. These geometric characteristics can be represented by Equation A.

 $Lf = Li \cdot (1 + Ex) \cdot (1 - Rs) \cdot (1 - Rq)$ (A)

Each one of these variables is influenced by material and process characteristics (factors). Qualitatively, the plant technician knows how a group of factors influence the final size of the tiles. Different product types have different critical control points<sup>[3,4]</sup>. For BIIA tiles, the factor to be controlled to produce dimensional uniformity is the linear firing shrinkage by controlling the bulk density and the maximum firing temperature. Other works in the literature agree with this statement<sup>[5,6]</sup>.

The proposal of this work is to study the dimensional variation, using mathematical modelling, and quantify the influence of a set of factors under the final size of ceramic unglazed tiles produced by single firing and wet milling route. Once the model is established, verification and evaluation of new control levels is possible and the efficiency of each case can be estimated.

#### 2. MATHEMATICAL MODEL

The model was build based on the theory of the maximum uncertainty of measurement, using a Taylor's series expansion. It takes into account 6 factors assumed independent: size distribution and water content of the atomised powder; compacting pressure; extraction time; mass of the tile and maximum firing temperature. These factors were chosen because of their importance, according to the literature and observation of manufacturing process. Each term of the series was determined by polynomial regression to experimental data, laboratory and industrial experiments. The model is presented from equations (B) to (E). The factors were considered independent, since in process they are individually controlled. The values of partial derivatives can be a constant or a function.

$$Ex = f(P)\Big|_{\overline{E},\overline{M},\overline{U}} + \Delta E \cdot \frac{\partial Ex}{\partial E}\Big|_{\overline{P},\overline{M},\overline{U}} + \Delta M \cdot \frac{\partial Ex}{\partial M}\Big|_{\overline{E},\overline{P},\overline{U}} + \Delta U \cdot \frac{\partial Ex}{\partial U}\Big|_{\overline{E},\overline{P},\overline{M}}$$
(B)

$$Rs = f(P)\Big|_{\overline{E},\overline{M},\overline{U}} + \Delta E \cdot \frac{\partial Rs}{\partial E}\Big|_{\overline{P},\overline{M},\overline{U}} + \Delta M \cdot \frac{\partial Rs}{\partial M}\Big|_{\overline{E},\overline{P},\overline{U}} + \Delta U \cdot \frac{\partial Rs}{\partial U}\Big|_{\overline{E},\overline{P},\overline{M}} \tag{C}$$

$$Rq = f(D) \bigg|_{\overline{T}, \overline{M}, \overline{E}} + \Delta T \cdot \frac{\partial Rq}{\partial T} \bigg|_{\overline{D}, \overline{M}, \overline{E}} + \Delta M \cdot \frac{\partial Rq}{\partial M} \bigg|_{\overline{D}, \overline{T}, \overline{E}} + \Delta E \cdot \frac{\partial Rq}{\partial E} \bigg|_{\overline{D}, \overline{T}, \overline{M}}$$
(D)

$$D = f(G)\Big|_{\overline{P},\overline{U}} + \Delta P \cdot \frac{\partial D}{\partial P}\Big|_{\overline{G},\overline{U}} + \Delta U \cdot \frac{\partial D}{\partial U}\Big|_{\overline{G},\overline{P}}$$
(E)

$$\Delta E = (E - \overline{E}); \Delta M = (M - \overline{M}); \Delta U = (U - \overline{U});...$$
(F)

where:

E = extraction time; M = tile mass; U = humidity of the atomised powder; P = compacting pressure; G = mean particle size of the atomised powder; D = bulk density; T = maximum firing temperature.

# 3. **RESULTS AND DISCUSSION**

Once the experimental parameters were determined by laboratory and industrial experiments, it was possible to evaluate the contribution of each factor on the variation of the final length of ceramic tiles. This evaluation was carried out considering the intervals of variation from each factor according to historical data of two plants. The results can be observed in Figure 1. In both plants the factors tile mass, firing temperature and water content were, in this sequence, the main factors responsible for the dimensional variation. The mean particle size of plant B has higher influence than compacting pressure; in plant A, the effect is the opposite.

Figure 1. Contribution of each factor on the variation of the final length of ceramics tiles.

### 4. CONCLUSIONS

The developed mathematical model could be applied in the study of dimensional variation of ceramic tiles. Mathematically, it can be written using an infinite number of independent factors. The factors considered can be individually controlled in a fabrication process.

The model allowed the experimental design to be carried out individually for each factor. In this way, a significant reduction in the number of runs was necessary for the determination of the experimental parameters. The simplification didn't affect the reliability of the results. This was possible mainly because the system is evaluated by numerical simulation, inside of narrow response intervals. The model does not hinder full factorial design to be applied. The regression of experimental data was satisfactorily done by first and second order polynomials. The runs in processing plants were important to assure the reliability of results. For the factors water content, size distribution and firing temperature, which were evaluated in laboratory conditions, the results were equally reliable.

The result of the numerical simulation showed narrower intervals of dimensional variation than the observed in the industrial reality for the following reasons:

- The simulation did not take into account feed interruption of the oven, what frequently occurs and causes high variations in the maximum firing temperature.
- The influence of the factors categorized as "others" could have been underestimated.
- The management system of production was not taken into account, from the system of data acquisition to the actions of control.
- The model does not consider the occurrence of out of form problems
- The experimental parameters of linear firing shrinkage were obtained with unglazed tiles.

Although with these limitations, the model was capable to differentiate the influence of chosen factors on the simulated conditions. These results showed that variations in the tile mass, maximum firing temperature and water content of the atomised powder are, in this order, mainly responsible for the dimensional variation in the studied process. This means that the current control limits of the tile mass are far away from the limits that represent an adequate situation.

The simulation showed that by restricting the control limits of the factors the process quality is increased. This increase occurs more efficiently when all the factors were simultaneously restricted. It can be also observed that these limits must be closer for 400x400 mm than 300x300 mm tile size. The definition of ideal limits depends on the tile size and the characteristics of each process, but it can be said that variations higher than 50g in tile mass, 2°C in firing temperature, 0.5% in water content of atomised powder and 0.8 MPa in compacting pressure increase in a significant level the variation of the final size of a ceramic tile As consequence, the quality level is reduced. For size distribution, it is desirable that the fractions do not undergo variations higher than 20%. The extraction time presented minor influence on size variation.

Once the model has been established, is possible to study and to plan different forms of management of the process to handle eventual economic and/or technical issues to keep the process operating in narrower intervals than the current levels. Process control is an activity that demands resources, mainly in the automation area. However, a great part of the control in the ceramic tile industry still depends on operations carried out by machines operator. In this way, all the cycle, from the collection of information to the control action, is subject to problems that result in loss of quality. This reality suggests a training program, awareness and incentives by management staff. These are also important tools that increase the efficiency of control actions.

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