STRESSES IN THE COMPOSITE SYSTEM: TILE – FIXING MORTAR – BASE

Dr. Josef K. Felixberger

State University of St Petersburg (IFMO)

Degussa. AG Construction Chemicals Europe
Head of Application Technology

ABSTRACT

In practice, when ceramic tiles are fixed on fresh screeds or bases, radiant flooring, or in balcony and terrace areas, detachments of the ceramic tiling can occur owing to the use of tile fixing mortars that are too rigid. Shrinkage of the base (hydration of the cement) or the longitudinal modification of the ceramic tiling as a result of thermal effects (solar radiation) can cause extreme shear stresses to develop in the adhesive layer, which can ultimately lead to detachments.

With the help of a physical model the distribution of the shear stresses in the mortar layer can be predicted and visualised. The shear stress peaks (τ_{Kl-max}) of the fixing mortar in the corners of the floor tile are of great interest, since a cementitious adhesive does not withstand shear stress peaks exceeding 0.5 N/mm² indefinitely.

The mathematical solution

$$\tau_{Kl\text{-}max}\left(l_{Fl}, b_{Fl}\right) = \varepsilon_U * \sqrt{\frac{E_{Fl} * d_{Fl} * G_{Kl}}{d_{Kl}}} * \tanh\sqrt{\frac{\left(l_{Fl}^2 + b_{Fl}^2\right) * G_{Kl}}{d_{Kl} * d_{Fl} * E_{Fl}}}$$

shows the influence of

- *the deformation of the base due to shrinkage or thermal expansion of the ceramic tiling* (ε_u)
- *the stiffness of the ceramic tiling* (E_{Fl})
- *the thickness of the mortar layer* (d_{Kl})
- *the shear stiffness of the cementitious adhesive* (G_{Kl}) *and*
- *the dimensions of the ceramic tile* (I_{Fl}, b_{Fl}, d_{Fl})

on the maximum shear stress (τ_{Kl-max}) *in the mortar layer.*

1. INTRODUCTION

In the 1970s it was discovered that thin-bed tile installation, with the use of a purely cementitious adhesive, could lead to damage in the ceramic tiling if, owing to longitudinal modifications of the base or of the ceramic tiling, excessively high stresses formed in the fixing mortar [1, 2].

By adhering the complete surface of a tile to the base, the forces resulting from the deformations of the base can be transmitted through the adhesive to the tiling and vice versa.

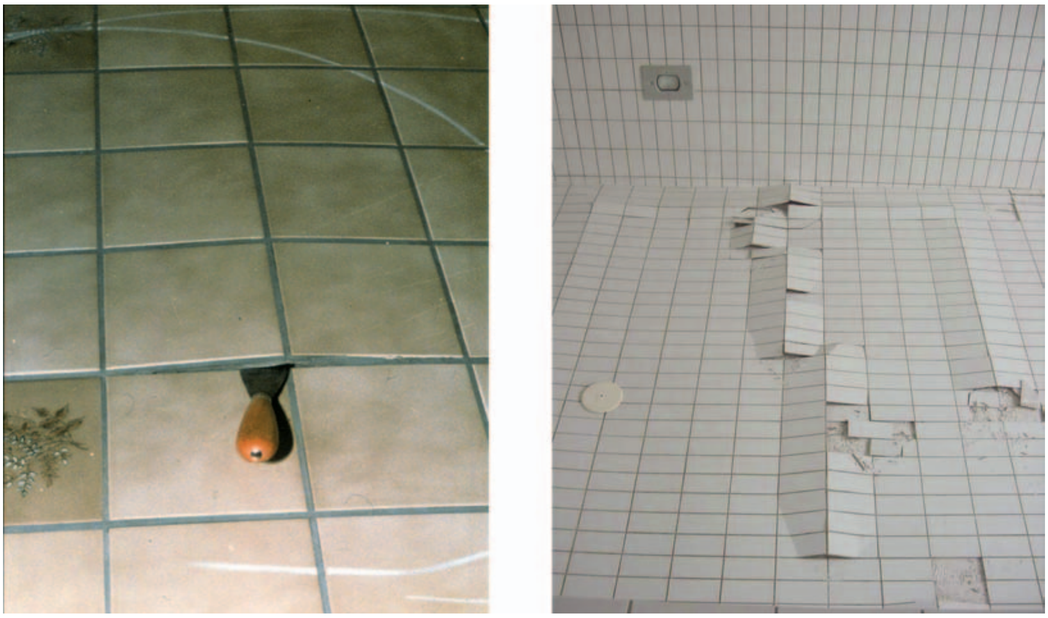


Figure 1. Detachment of ceramic tilings owing to the formation of excessively high stresses

2. CAUSES OF THE STRESSES

What can cause the relative movements between the base and the ceramic tiling? On the one hand to be mentioned are the different expansions (Δl) that materials undergo with variations in temperature. This longitudinal modification becomes greater, in absolute form, with the increase in the variation of temperature (ΔT), the initial length of the piece (l_0) and the specific coefficient of linear expansion (α) of the material [3].

$$\Delta I = I_0 * \alpha * \Delta T$$

	α [mm/mK]
Ceramic tile	0,006
Anhydrite base	0,008
Cement base	0,012
Chipboard	0,030

Δl	Absolute longitudinal modification	[mm]
l_0	Initial length	[m]
α	Coefficient of linear expansion	[mm/mK]
ΔT	Variation of temperature	[K]

Table 1. Examples of coefficients of linear expansion [4]

When, for example, the midday sun raises the temperature of the ceramic tiling of a 5 m (l_0) wide terrace by 40ºC (ΔT), this tiling tends to expand 1.2 mm (Δl). Owing to the composite construction and since, if the base is heated, it does so more slowly, the tiling, adhesive and the base will need to absorb the resulting stresses.

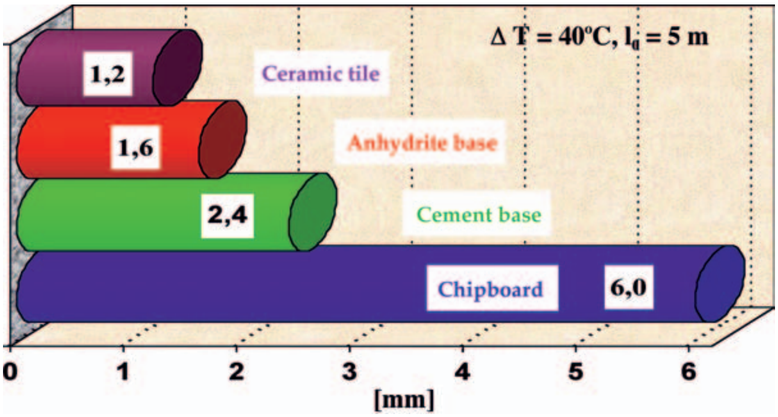
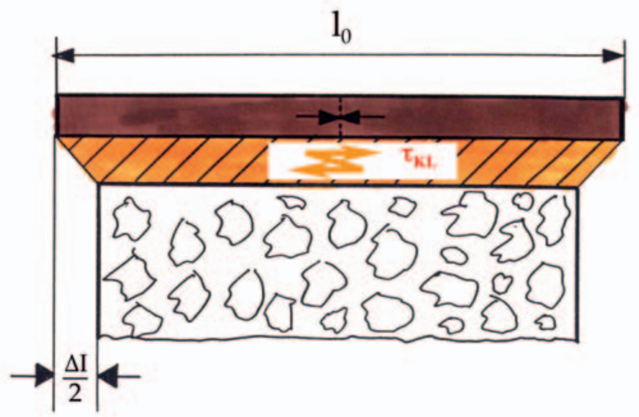


Figure 2. Absolute longitudinal expansion of a terrace tiling

A second group of causes of the relative movements between the tiling and the base are the time-dependent shrinkage processes, caused by the setting of the screed or the concrete base. Depending on the composition of the concrete and on the environment, this residual shrinkage can, over the years, reach approximately 1 mm per metre [5].

The magnitude of the deformation of the base is indicated in the literature as an adimensional measurement index (ε_U), which establishes a relation between the longitudinal modification of the base (Δl) and the initial length (l_0) [3].

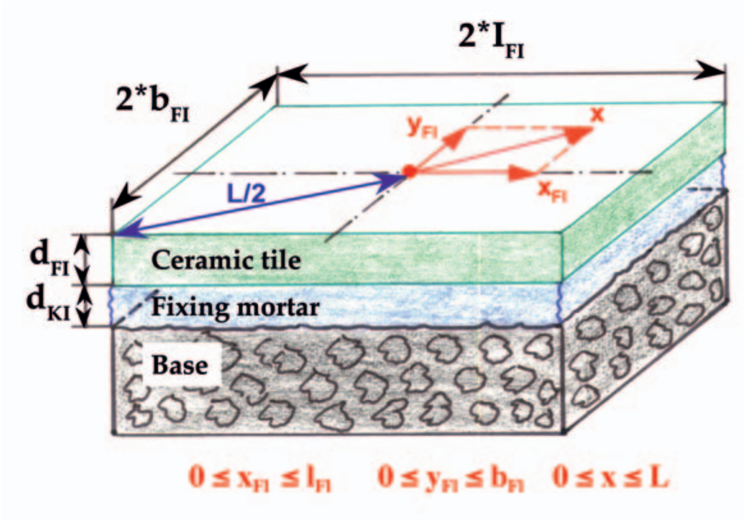
$$\varepsilon_U = \frac{\Delta l}{l_0} \qquad (1)$$



Δl	Absolute shrinkage	[mm]
l_0	Initial length of the base	[mm]
ε_U	Deformation of the base	[mm/mm]

Figure 3. Shortening of the base owing to cement hydration

3. MATHEMATICAL MODEL



d_{FI}	Thickness of the tile	[mm]
d_{KI}	Thickness of the mortar layer	[mm]
x	Radial distance from the centre of the tile	[mm]
x_{FI}	Distance from the centre of the tile in the longitudinal direction	[mm]
y_{FI}	Distance from the centre of the tile in the transverse direction	[mm]
$L/2$	Maximum radial distance from the centre of the tile	[mm]
l_{FI}	Half tile length	[mm]
b_{FI}	Half tile width	[mm]

Figure 4. Designation of the parameters

A step by step deduction of the mathematical relations would exceed the frame of the present paper and can be consulted in [6].

In this fundamental deduction it is possible to set out from equilibrium between the compression force in the tile (F_{FI}) and the shear force in the fixing mortar (F_{KI}).

(2) $$F_{FI}(x) = -F_{kl}(x)$$

Expressing the compression force in the tile and the shear force in the fixing mortar in relation to the compressive stress (σ_{FI}) and the shear stress (τ_{KI}), respectively, gives equation (3) [7].

(3) $$2 * b_{FI} * d_{FI} * \sigma_{FI}(x) = -2 * b_{KI} * \tau_{KI}(x) * x$$

Taking into account the differential modifications of the shear stress, of the compressive stress and of the compression of the tile by means of the differentiation in accordance with the radial distance x from the centre of the tile, after some transformation, gives the following non-homogeneous second-order differential equation with constant coefficients (4):

(4) $$\frac{d_{KI} * d_{FI} * E_{FI}}{G_{KI}} * \frac{d^2}{dx^2}\sigma_{FI}(x) - \sigma_{FI}(x) = -\varepsilon_U * E_{FI}$$

As a closed solution of the system of differential equations for the shear stress distribution equation (5) is obtained [8].

$$(5) \qquad \tau_{KI}(x_{FI}, y_{FI}) = \varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * G_{KI}}{d_{KI}}} * \frac{\sinh\sqrt{\frac{(x_{FI}^2 + y_{FI}^2) * G_{KI}}{d_{KI} * d_{FI} * E_{FI}}}}{\cosh\sqrt{\frac{(l_{FI}^2 + b_{FI}^2) * G_{KI}}{d_{KI} * d_{FI} * E_{FI}}}}$$

4. DISCUSSION OF THE SHEAR STRESS FORMULA

4.1. GENERAL

In the corners of the tile, the shear stress must acquire maximum values, since the value of the relative movement between the ceramic slab and the base maximises there. Setting x_{FI} and y_{FI} equal to l_{FI} and b_{FI}, i.e. positioning oneself in the corner of the tile, for the maximum shear stress in the corners of the slab expression (6) is obtained:

$$(6) \qquad \tau_{KI\text{-}max}(l_{FI}, b_{FI}) = \varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * G_{KI}}{d_{KI}}} * \tanh\sqrt{\frac{(l_{FI}^2 + b_{FI}^2) * G_{KI}}{d_{KI} * d_{FI} * E_{FI}}}$$

Therefore, the maximum shear stress in the fixing mortar in the corners of the tiles ($\tau_{KI\text{-}max}$) depends exclusively on the geometric factors tile length (l_{FI}), tile width (b_{FI}), tile thickness (d_{FI}), adhesive layer thickness (d_{KI}), on the material properties modulus of elasticity of the tile (E_{FI}), shear stiffness of the fixing mortar (G_{KI}) and on the relative movement of the ceramic tiles with regard to the base (ε_U).

4.2. INFLUENCE OF THE SHEAR MODULUS OF THE FIXING MORTAR (G_{KI})

If typical values are set for the individual parameters and the shear stress is calculated for increasing distances from the centre of the tile, the distribution represented in figure 5 is obtained.

While a porcelain tile has an assured modulus of elasticity of 35,000 N/mm^2 [9], the shear modulus of the fixing mortar G_{KI} must be chosen freely since no values are available in the literature. A cementitious tile adhesive would not withstand a maximum shear stress peak of 2 N/mm^2 indefinitely in the corners of the tile. The adhesive would disaggregate beginning in the corners and detachments of the ceramic tiling would occur. The detachments would propagate inwards until the radial shear stress had a value below 0.5 N/mm^2. If the adhesive was extremely rigid, it might then be left with just a single central adhesion point, whose adhesion surface area might then be insufficient to support the tile's own weight on a vertical wall.

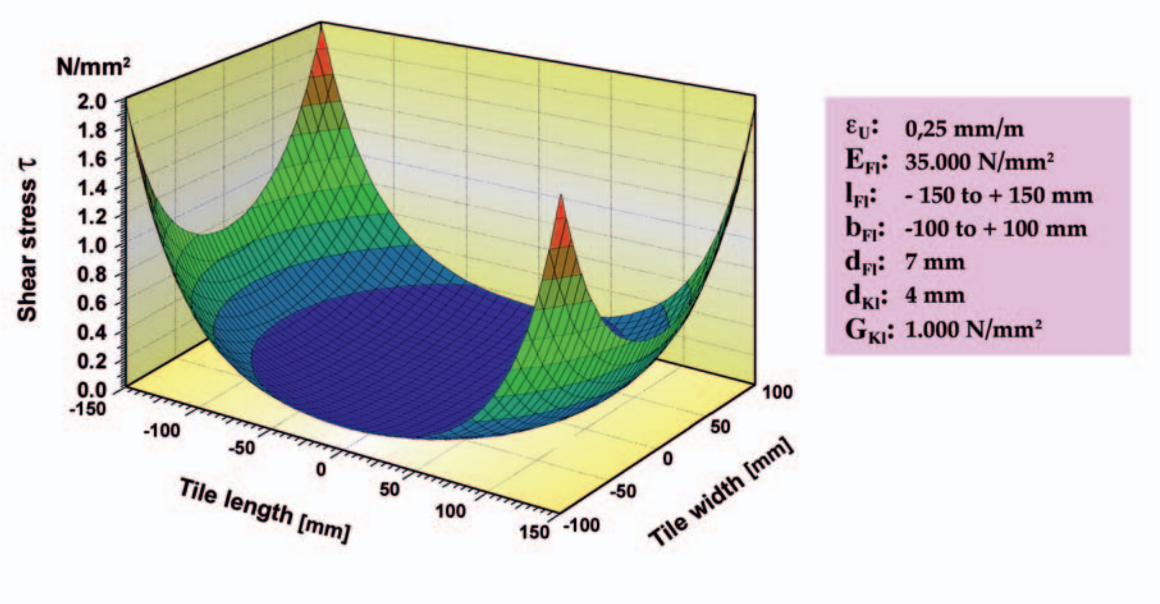


Figure 5. Radial distribution of the shear stress for a rigid fixing mortar

The reduction of the shear modulus from 1000 to 100 N/mm² leads to a significantly smaller shear stress in the mortar, since the fixing mortar yields more and is more flexible and, owing to the low internal strength of the adhesive, can only generate a significantly smaller stress.

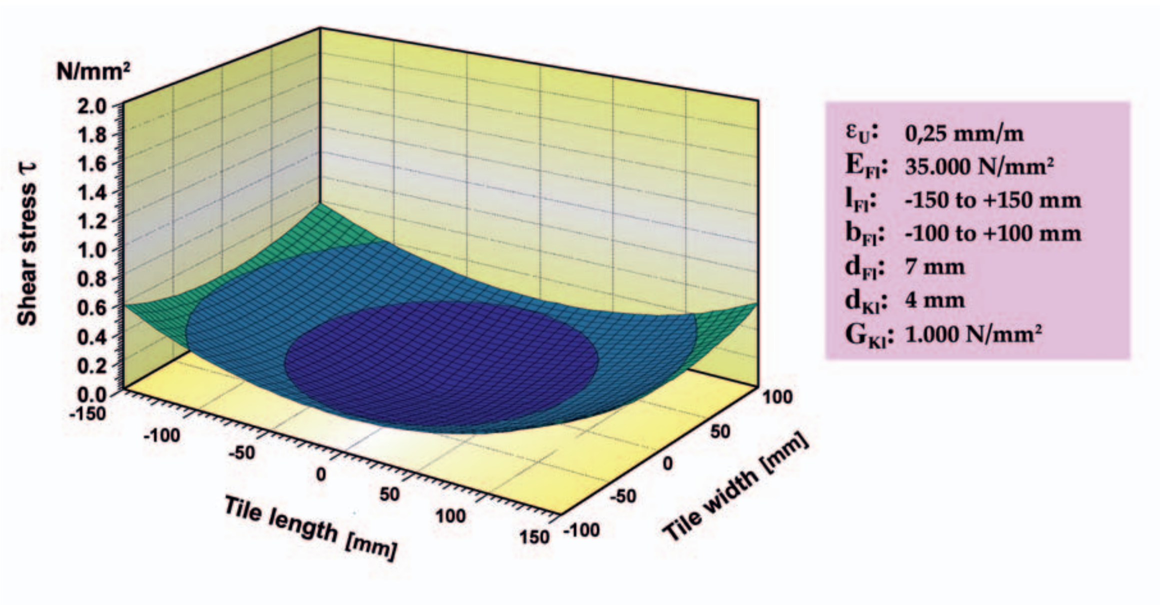


Figure 6. Distribution of the shear stress for a flexible fixing mortar

4.3 INFLUENCE OF THE RELATIVE MOVEMENT OF THE CERAMIC TILE WITH REGARD TO THE BASE (ε_U).

The following diagram displays the maximum shear stress peaks for a porcelain tile of 20 x 30 cm, simultaneously varying the shear modulus of the adhesive (G_{KI}) and the relative movement between the ceramic tile and the base (ε_U).

Two limit cases need to be discussed. In the case that the adhesive is infinitely elastic, i.e. $G_{KI} = 0$, no shear stress forms in the adhesive, even in the case of a great deformation of the base, since the adhesive acts in a certain sense as an ideal separation layer. In the case that there is no relative movement between the base and the ceramic tile ($\varepsilon_U = 0$), no shear stress develops either, independently of the stiffness of the fixing mortar. Especially critical shear stress peaks form in the corners of the tile when, in the case of the appearance of high residual shrinkage in the base, a very rigid fixing mortar is used (low dispersion powder content). In this case, the stress peaks in the corners of the tiles are in the red area of the diagram, which inevitably leads to the destruction of the cementitious adhesive.

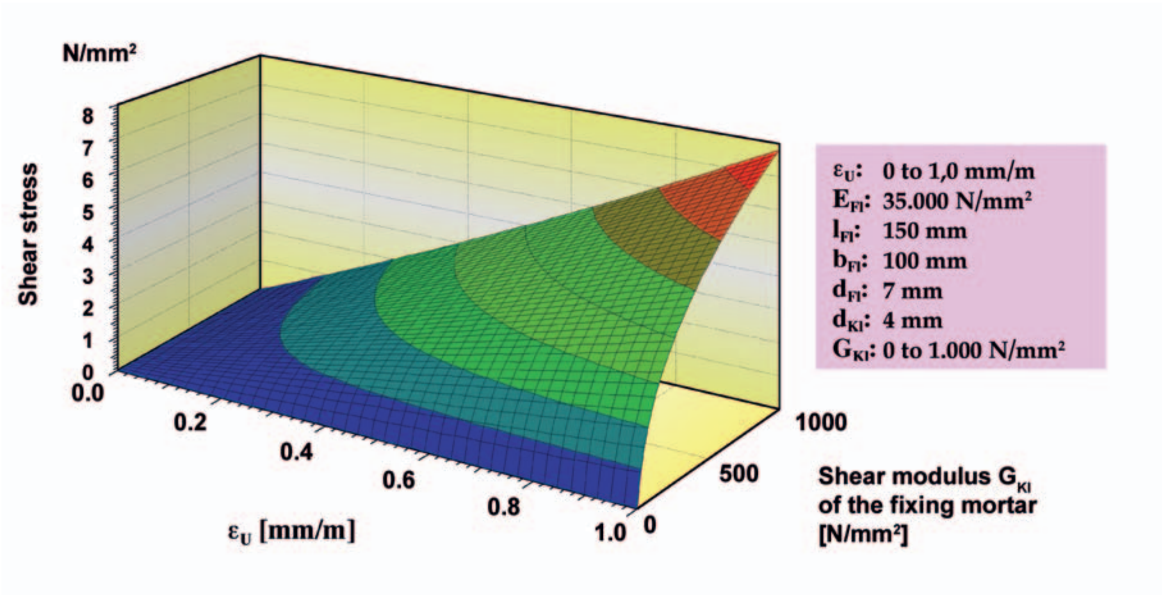


Figure 7. Influence of base deformation and shear modulus of the cementitious adhesive on the maximum shear stress in the corners of the tile

4.4. INFLUENCE OF TILE SIZE (l_{Fl}, b_{Fl})

Professionals often allege that shear stress rises considerably when tile size increases. Equation 6 shows that this is only relatively true. For the extreme case of an infinitely large tile the subradical expression converges as argument of the hyperbolic tangent towards infinite [8]. The hyperbolic tangent of an infinite number is 1, so that the maximum shear stress in the corners of an infinite tile can be calculated from equation 7. It is interesting that this maximum value no longer depends on the dimension of the tile, but exclusively on the own stiffness of the tile, on the shear stiffness of the adhesive, on the thickness of the tile, on the thickness of the adhesive layer and on the relative movement (shrinkage of the base, thermal expansion of the ceramic tile) of the ceramic slab with regard to the base.

$$\tau_{KI-max}(\infty,\infty) = \varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * G_{KI}}{d_{KI}}} \tag{7}$$

Figure 8 shows how the maximum shear stress rises when tile size increases (l_{FI}, b_{FI}) in accordance with equation 6, to approach asymptotically a maximum value in accordance with equation 7. In the given case, for a mosaic with an edge length of 4.5 cm, maximum shear stress peaks of 0.2 N/mm² would form. At a tile size of more than 40 cm in edge length, the maximum shear stress of 0.62 N/mm² does not continue to increase in the corners of the tiles. However, for a large-sized tile, this means that the areas that are at a distance of more than 20 cm from the centre of the tile are subject to a maximum shear stress of 0.62 N/mm² and in time these areas will detach.

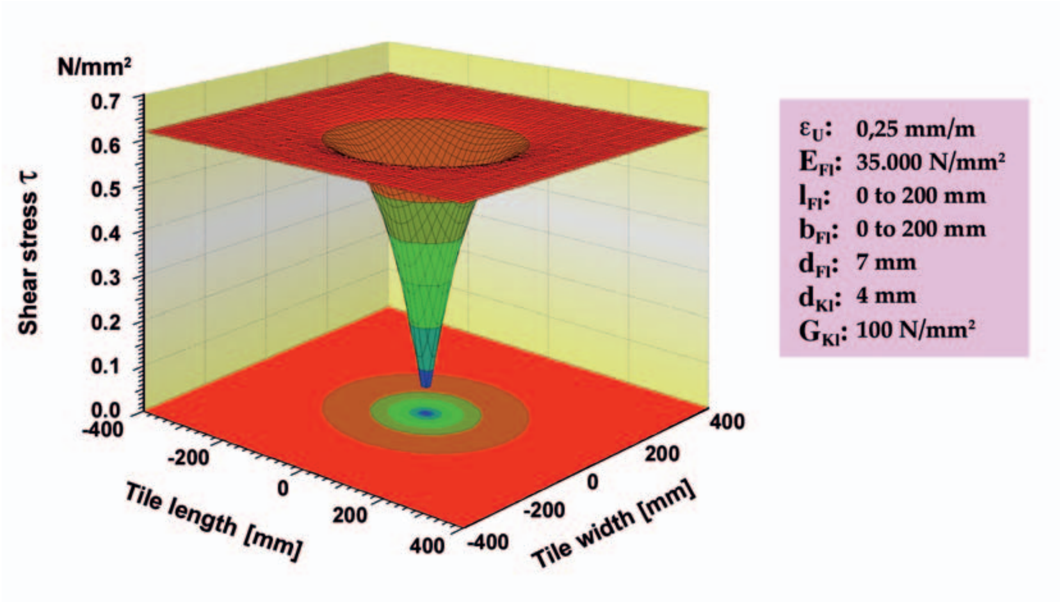


Figure 8. Maximum shear stress in the corners of the tile as a function of tile edge length

Figure 9 depicts another representation of the same circumstance. The curve indicates up to which tile size, as a function of the adhesive shear modulus (G_{KI}), the shear stress peaks in the corners of the tiles will not exceed a value of 0.5 N/mm², at which a safe and lasting tile installation may be assumed. As deformation of the base a value of 0.25 mm per metre base has been assumed. In the case of high shear moduli, i.e. in the case of rigid, inflexible, adhesives (G_{KI} = 1000 N/mm²), this value is already exceeded for edge lengths above 12 mm. In the case of soft, deformable adhesives, with a G modulus below 65 N/mm², the critical value of 0.5 N/mm² is not even reached for an infinite tile.

The development of the curve was calculated from equation (8):

$$l_{FI-max} = \varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * d_{KI}}{2 * G_{KI}}} * \text{artanh}\left[\frac{\tau_{KI-max}}{\varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * G_{KI}}{d_{KI}}}}\right] \tag{8}$$

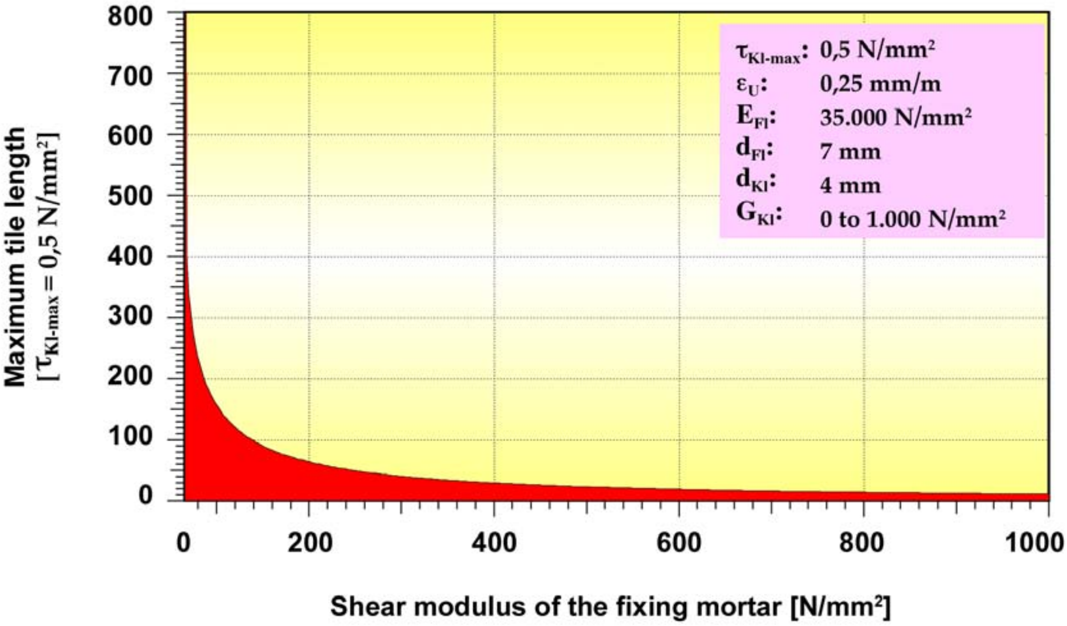


Figure 9. Maximum tile size as a function of adhesive shear modulus, when the maximum shear stress shall not exceed 0.5 N/mm²

4.5. INFLUENCE OF THE JOINTS

The magnitude of the maximum shear stress in the corners of the tile for an infinitely large tile is obtained from equation (7). However, since tiles have a finite size equation (6) must be used for the calculation of the maximum shear stress in the corners of the tile.

If a ceramic tiling is considered, this is made up of tiles and a grid of joints. The case is assumed in which the grid of joints is composed of a mortar grout that has the same stiffness, i.e. the same modulus of elasticity as the ceramic tiles. From a physical point of view, for the stresses, this would be identical to considering there was no joint, i.e. that the entire tiling was solely made up of one almost infinitely large ceramic slab. If the joints have not been filled or if this has been done with a very elastic joint filling, the actual dimensions of the tiles can be inserted in equation (6). The reason is that in the case of shortening of the base or of thermal heating of the tiling, the individual ceramic tiles can move relatively to each other without "disturbing" each other. Thus, the presence of the tile-to-tile joints reduces the maximum shear stress compared with a tiling without any joints. The absolute value of the reduction in shear stress in the corners of the tiles as a result of the joints, in comparison with a tiling without any joints, is calculated by subtracting equation (6) from equation (7).

$$\Delta\tau_{KI\text{-}max}\left(l_{FI}, b_{FI}\right) = \varepsilon_U * \sqrt{\frac{E_{FI} * d_{FI} * G_{KI}}{d_{KI}}} * \left[1 - \tanh\sqrt{\frac{\left(l_{FI}^2 + b_{FI}^2\right) * G_{KI}}{d_{KI} * d_{FI} * E_{FI}}}\right] \quad (9)$$

For a very small mosaic piece, i.e. l_{FI} and b_{FI} equal to zero, the hyperbolic tangent has a value equal to zero. In this way, the reduction in shear stress is maximum, i.e. no shear stress develops. In contrast, for an infinitely large tile, $l_{FI} = b_{FI} = \infty$ the hyperbolic tangent adopts the value 1 [8], i.e. no reduction in shear stress takes place.

Thus, the expression between parenthesis 1 - tanh practically represents a reduction factor (r):

$$r = 1 - \tanh\sqrt{\frac{\left(l_{FI}^2 + b_{FI}^2\right) * G_{KI}}{d_{KI} * d_{FI} * E_{FI}}} \quad (10)$$

of the maximum shear stress in the corners of the tiles, caused by the joints and the finite size of the tiles linked to these joints.

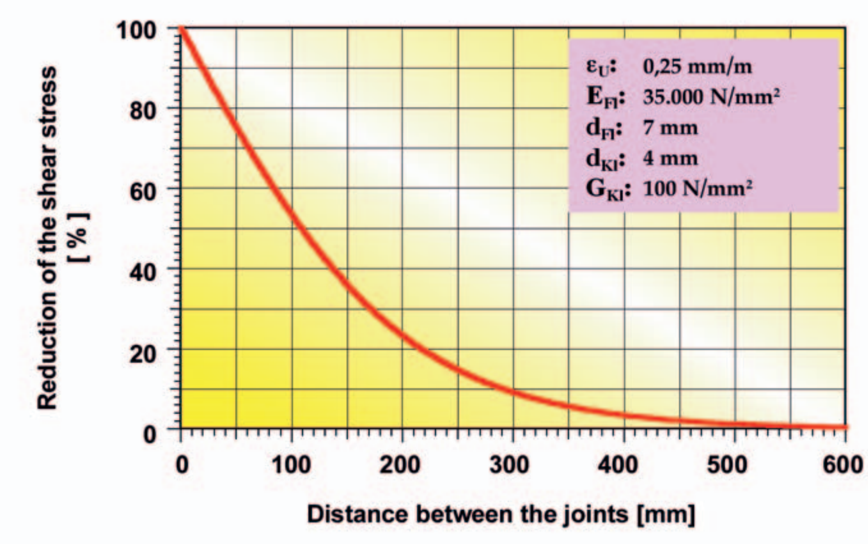


Figure 10. Reduction of maximum shear stress by the joints in the ceramic tiling

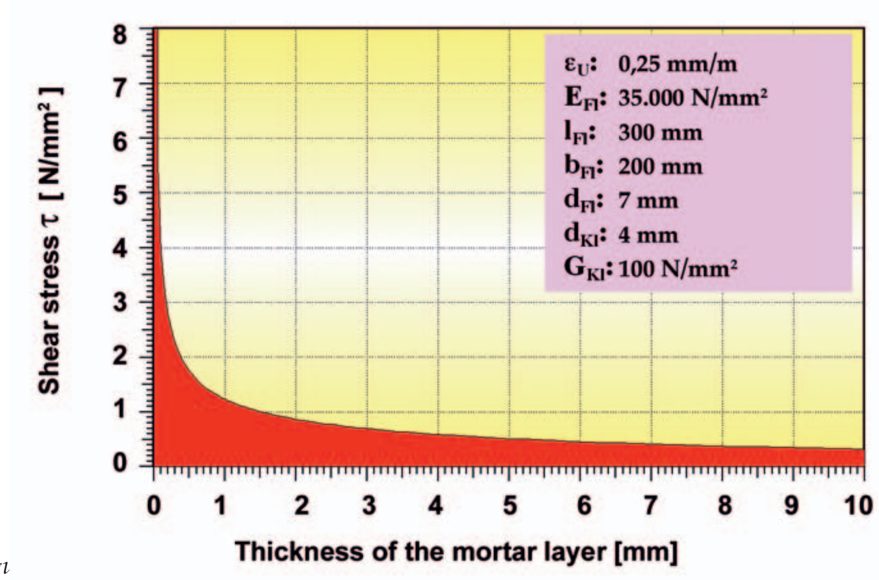
Figure 10 graphically represents the behaviour for a given tile and a defined fixing mortar. At a distance between joints of 5 cm (large mosaic), the maximum shear forces are reduced by 75% in the corners of the tiles owing to the joints. In the case of a distance between joints of 20 cm, the reduction in stress is still approximately 25%. Beyond a tile size of 50 cm there is no longer practically any reduction in the maximum shear stress in the corners of the tiles.

4.6. INFLUENCE OF ADHESIVE LAYER THICKNESS (d_{KI})

The experience in practice shows that the thickness of the adhesive layer can contribute significantly to reducing shear stress. If in equation 6 only the thickness of the adhesive layer (d_{KI}) is varied, then, in accordance with figure 11, this patently obvious truth is confirmed. The shear stress in the fixing mortar increases exponentially for an adhesive layer thickness below 1 mm. For an adhesive layer thickness exceeding 4 mm only a low reduction in shear stress is obtained.

5. SIGNIFICANCE IN PRACTICE

Equation (6) describes the maximum shear stress that develops in the corners of the tiles. If $\tau_{Kl\text{-}max}$ has a value exceeding 0.5 N/mm², the fixing mortar disaggregates in time. Thus, beginning in the corners of the tiles, detachments take place that propagate concentrically towards the centre of the tile, reducing the values of l_{Fl} and b_{Fl}. The detachment advances until, owing to the reduced values of l_{Fl} and b_{Fl}, the value of $\tau_{Kl\text{-}max}$ falls below 0.5 N/mm². Depending on how much of the fixing mortar surface area remains in contact, the own weight of the tile may be sufficient to cause it to fall off the wall. In outside environments, water can penetrate into the detached areas and destroy the ceramic tiling as a result of the pressure that builds up with frost.



Figı *ness*

Who can contribute to keeping the shear stress from growing too much?

5.1. PROPRIETOR

Equation (6) indicates that the maximum shear stress increases with tile size, tile stiffness (E_{Fl}) and tile thickness (d_{Fl}). A thicker tile is more difficult to compress than a thin one, as a result of which, given a deformation of the base, a higher shear stress is generated. The thickness and modulus of elasticity of the tile are determined by the manufacturing process and they can only vary to a minor extent. The proprietor chooses the tile size and size, in turn, is subject to fashion trends. Porcelain tile, chosen gladly for aesthetic reasons, at 35,000 N/mm² has a greater internal stiffness than stoneware tiles (E_{Fl} = 15,000 N/mm²).

Essentially, the magnitude of the deformation of the base or, in general, of the relative potential movement between the base and the ceramic tiling (ε_U) can also be influenced by the proprietor. The greater this relative movement, the greater will be $\tau_{Kl\text{-}max}$. The potential deformation of a concrete slab depends essentially on its residual moisture content, established by the CM method, and on the advance of cement hydration. For safe tile installation on a cement base, the literature requires a residual moisture content of 2% (CM method). For a cement base, this residual moisture content is presumed to be reached in the case of a normal climate (20ºC, 50% relative humidity) after 28 days [10]. If this length of time is unavailable or, as is observed frequently in practice, a moisture content of 2% is not reached according to the CM method even after 28 days, the progress of the building work can be accelerated by using decoupling layers, or using fixing mortars modified with synthetic materials.

The selection of the tiling to be installed also influences the possible relative movement between the tiling and the base. Figure 2 shows that chipboards with synthetic resins display a coefficient of thermal expansion five times greater than that of stoneware tiles [11]. In addition, it should be taken into account that black tiles heat and expand considerably more due to solar radiation that light-coloured tiles.

5.2. THE ELABORATION BY THE TILE FIXER

The tile fixer essentially has an influence on the parameters adhesive layer thickness (d_{Kl}) and fixing mortar shear modulus (G_{Kl}), corresponding to equation (6). Below a certain adhesive layer thickness, the maximum shear stress increases exponentially. For this reason it is important that the minimum quantities recommended by the manufacturer should be used with regard to primers, bond bridges and fixing mortar over the entire surface.

The tile fixer can influence the stiffness, i.e. the shear modulus of the fixing mortar (G_{Kl}), by means of the corresponding selection of the fixing mortar or, respectively, by the addition of synthetic material in dispersion. Mortars modified with synthetic materials display a smaller resistance to deformation by shear than the purely cement-based adhesives. Thus, shear stresses can effectively be reduced (figures 5, 6 and 7). The use of fixing mortars modified with synthetic materials is especially advantageous in the case of fixing large-sized slabs on fresh cement bases, in the case of tile installation on radiant flooring or in the case of surfaces subject to strong solar radiation.

In practice, it is often forgotten that by filling joints with a hard grout mortar, the length (l_{Fl}) and the width (b_{Fl}) of the tile becomes practically identical to the dimensions of the whole tiled area. If the possibility existed of carrying out the grouting some weeks after the tile installation, parameters l_{Fl} and b_{Fl} in equation 6 would really be limited to the boundaries of the tile. The base could dry quickly through the open joints and the individual tiles could move freely and, therefore, generate smaller shear stresses (figure 10).

5.3. THE CHEMICAL INDUSTRY FOR BUILDING CONSTRUCTION

The manufacturers of fixing mortars can only influence the shear modulus (G_{Kl}) of the fixing mortar. Specifically, the resistance to deformation by shear of the cementitious fixing mortar can be varied by the appropriate selection of the dispersion or of the dispersion powder and their quantities (figures 5, 6, 7).

At present few or no data are available on cementitious fixing mortars because the measurement of the shear modulus is relatively complicated [12] and inexact.

6. LIMITS OF THE MODEL

The most decisive limitation of the model resides in the fact that it is only valid for the Hooke elastic area. Therefore, the reduction of stresses due to the plastic deformation of the fixing mortar cannot be described. However, in practice it is known that due to the permanent action of the shear forces, beyond a certain magnitude, concrete slab, bases or screeds, fixing mortars, etc. can reduce stresses by means of creep [13, 14, 15].

Therefore, the present elastic considerations of the composite bond represent a worst-case scenario. The model enables describing how each parameter tends to influence the composite system in an individual way.

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