INTRODUCTION TO A CONCEPTION OF SIMPLIFIED MODELS IN THE QUANTITATIVE ESTIMATION OF STRESSES IN CERAMIC TILINGS (CASE: DECK-ADHESIVE-FLOOR SYSTEM. MESES)

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ABSTRACT

Excellent studies, articles and communications are available that approach in concrete form the issue of ceramic tilings, although studies that analyse these from a qualitative approach are much more prolific, establishing as causes various factors which, together with the materials, are usually synthesised in a group that includes design, execution and environment, presenting certain suggestions as advice for the common technician, thus completing the tight basic criteria of recognised manuals and guides, like the Guía de Baldosas Cerámicas (Ceramic Tile Guide).

However the problem becomes more pronounced when we try to find evaluation techniques that estimate the problem in a quantitative way, i.e. in numerical form.

The present paper seeks to establish, insisting again on the need for quantification, a possible line for estimating the basic stresses on the ceramic tile, within the technician's usual field of knowledge; the paper is the result of a teaching study assignment [1].

1. INTRODUCTION

First of all, it should be noted that the study undertaken does not focus on characteristics which, on a qualitative level, occur in ceramic tiling pathologies. Excellent works are available in this respect from which we have concluded, synthesised and taken the references cited, which is why for greater detail we suggest they be consulted; a sample is given. [3]-[13]

Thus, the present paper fundamentally wishes to insist – by a first contribution – on the need to generate studies that consider quantitative approaches in accordance with the qualitative basis available in the ceramic sector.

A first approach to the problem indicates the existing difficulty since, if we try to limit the number of variables that play a role, we find ourselves faced with the task of distinguishing between the qualitative effect and the variable that represents this univocally with certain accuracy. At most we can establish that the observed anomalies are related to the characteristics of the substrates and the adhesive, the arrangement and size of the joints, and the size of the ceramic tile, assuming, to start with, that their selection and installation have been appropriate, as these last two causes are not quantifiable, consequently leading to an impracticable complexity when it comes to attempting to determine a model that combines these causes quantitatively.

A more explicit analysis has led the experts to think that the essential focus should lie on the issue that the stresses and strains between substrate-adhesive-tiling are not well known. This approach has given rise to current studies, which analyse these evolving stresses and strains, essentially based on modelling with methods like the FEM (finite element method), which are at present a little sophisticated for the common technician and which, on the other hand, in view of the difficulty, require simplifications of the model, geometries and states of the actions.

In this context, given the quantitative introductory character of our subject, the present paper seeks to address the problem from an approach closer to the common technician, which has traditionally provided good results, namely that of the simplified methods, where the technician's basic knowledge is used. A first modelling is thus proposed, for the usual case of the deck-adhesive-floor system, called MESES.

MESES, corresponding to the Spanish for Simplified Method of Structure-Floor Estimation, seeks to evaluate the most unfavourable normal maximum stress that the tile undergoes, and provide an orientational estimation for the adhesive and the contact layer with the deck, with an acceptable error from the safety standpoint.

The method has been generated from a preliminary approach, establishing a model to analyse the increase in the strains produced by any variation, and obtaining, by means of the simple hypotheses used to develop the model, a simple formulation.

The results obtained, in a first comparative verification with FEM modelling for stresses caused by an increase in temperature, can be considered acceptable, while at the same time, the formulation of MESES explains some characteristics that were only suggested by the FEM, acting as a mutual complement. (For this verification, the study [2] has been used as a reference, given the difficulty of having experimental data, which are practically non-existent).

Finally a plan is outlined for future work, which combines this type of model jointly with real and experimental data, complemented by the most advanced methods.

2. PRESENT QUANTITATIVE SITUATION OF THE DECK ADHESIVE-FLOOR SYSTEM. BASE REFERENCE

The introduction indicates the convenience of minimally outlining the situation of the problem involved, and of the proven Base Reference for comparison, to facilitate the subsequently adopted analysis.

The present quantitative situation of the system (comprising the deck, as fixing background, the adhesive layer, the ceramic tiles or floor, and the tile-to-tile joints), which for the sake of simplification we shall call the Deck/Floor System, is in a primary state given two circumstances inherent to the problem:

1. The difficulty of discerning and delimiting the precise variables, as well as their reach, which represent the problem. Basically we speak of stresses, the result of restricted deformations, but, what are the parameters and/or essential variables that play a role, how and to what degree?
2. The difficulty of establishing a general model that provides a basic approach, acceptable for analysis and yet accessible for the common technician.

An attempt to respond concisely to both points would not only be impracticable (the definition itself of all the possible variables would already represent a handicap, i.e. shrinkage, porosity, temperature, composition of the, size of the pieces, mechanical and-chemical characteristics of the adhesive…, which, together with the geometry, structural ties…, of the deck represent a high complexity), but also operationally unfeasible.

In view of this situation, and since the quantification is essentially subordinated to two possible lines, that of formulation and/or numerical modelling, and the experimental line, we will work along the former (not just because the experimental line requires instruments and techniques that are not very accessible, but also because their results are limited to the type of tested materials, and become obsolete when these change their characteristics).

In the chosen line, at the moment, work is conducted practically in models deriving from numerical analysis, particularly the finite element method, FEM, since the formulation or formal resolution of the Deck/Floor System involves accounting for its geometric configuration, i.e. solving the problem of a slab, an aspect that has only been solved theoretically in particular cases (depending on boundary conditions).

For example, the elastic strains of an isotropic slab with uniformly distributed load, q, at right angles to its plane, responds to the classic Lagrange equation:

$$\partial^4\delta / \partial X^4 + 2 \cdot [\partial^4\delta / (\partial X^2 \partial Y^2)] + \partial^4\delta / \partial Y^4 = q / D \quad ; \quad D = \text{Bending stiffness of the slab.}$$

(Its formal solution requires trigonometric series). The appearance of computers has fostered the appearance of numerical methods, like FEM, which entail a certain difficulty for many technicians.

To the situation described is to be added the lack of a database, with particular levels and values, on the innumerable daily cases that occur of cracking, buckling and/or detachment of the floor.

The foregoing justifies why we need to go to a proven quantitative reference, in view of this present state, which serves as a basis for analysis and subsequent comparison.

Among the excellent articles in this respect, we have selected the one [2] that, while topical, simultaneously presents verified results as well as a close reflection of the application of the FEM, which we shall term the Base Reference hereinafter.

3. ANALYSIS FOR SIMPLE GENERIC MODELLING. PREMISES AND PARAMETERS.

If the FEM applied to the Deck/Floor System is minimally analysed, even though it is one of the most powerful methods, it currently poses some problems, such as:

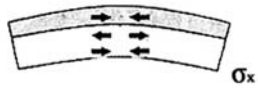
- Complicated characterisation (selection of the FE, and number of these).
- Present need for simplicity of the set (simplification of the Deck…).

These two characteristics, together with the difficulty for use by the technician, suggest the possibility of obtaining more basic, simplified, models that provide a first estimation, since in general the technician essentially uses maximum values (of stress and/or deformation), in his criteria of selection and dimensioning.

An approach of this type, which leads us to a simplified method that estimates the maximum stresses in the Deck-Adhesive-Floor system (in this study, restricted to normal stresses), requires generic modelling of the simplest possible type. This means that the deck cannot be treated as a slab, because this would irremediably take us back again to the previous section, i.e. we must consider a feasible geometric simplification of this, while simultaneously determining the parameters-variables essentially acting in the system, as an initial approach to the model. Thus, based on an analysis of content and geometrics, the following set of initial premises and parameters may be considered essential.

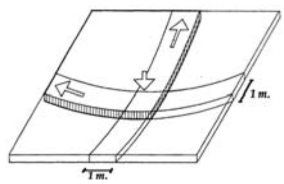
3.1. PREMISES

P1.- Determination of the value of the most unfavourable stress that occurs in the Deck/Floor System, in this case σ_x.



Normal stresses

P2.- Considering the simplification of the deck. It can be assimilated, for example, to a border or beam, if it is uniaxial, or to two perpendiculars if it is bidirectional, of a metre wide, a simplification methodology that already exists [16] (with an error, in regard to safety, 17-26%).

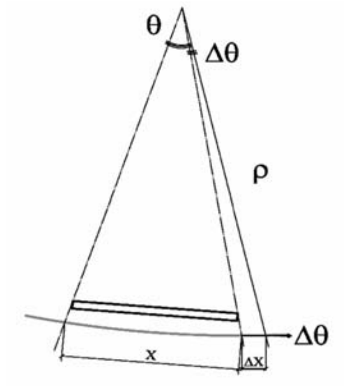


Adapted from [14]

P3.- Adoption of the classic elastic beam model, according to P2, and the Euler-Bernoulli curvature study, which includes the hypothesis of the flat sections, where:

$$1/\rho = \Delta\theta / \Delta x \approx M / EI$$

(M, flector moment in the cross-section)



P4.- Compatibility condition of the System. The compatibility condition includes:

- Compatibility of cause distribution. (E.g. that the mean temperature increase, or another cause, of a whole area or part of it, is equivalent to the mean increase undergone by the floor tiles, $\Delta T \approx \Delta T_{\text{tiles}}$).
- Compatibility of the total deformation with the partial ones, independently of the causes (E.g. that $\Sigma_{\text{deformations}}$ by temperature or another cause, of a whole area or part of it, is equivalent to the sum of the deformations of all the tiles and existing joints).

3.2. ESSENTIAL PARAMETER

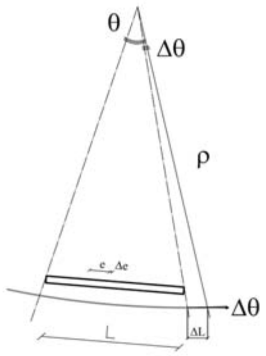
From premises P2 and P3 it is inferred that the essential parameter, in the classic study of deformation analysed, is the turn of the considered cross-sections, θ , which increase, $\Delta\theta$, with any incidental variation in the system, and since in this system (deck-adhesive-floor), the dimensional variations of their magnitudes (whether by temperature, shrinkage, moisture…, assumed good selection and installation of the tiles and adhesive), generally involve stresses deriving from the restriction of the ensuing strains, we can then consider that $\Delta\theta$ is a function of length, L, and position x, according to the adopted deck simplification. Thus:

$$\Delta\theta\ (L, x) = (\partial\theta/\partial L) * \Delta L + (\partial\theta/\partial x) * \Delta x$$ (ΔL, Δx, depend on temperature, shrinkage, ...)

Observation: In [1], a more complete analysis is made of this type of increase, $\Delta\theta$, together with the vertical displacement, $\Delta\delta$, which is not considered.

4. FORMULATION OF MESES. SIMPLIFIED METHOD OF STRUCTURE-FLOOR ESTIMATION

Combining the premises set out above, together with the equation for the increase of turn, $\Delta\theta$, according to the previous section, we obtain the basic formulation of MESES for normal stress, σ_x, in the tile, which for convenience we will designate σ_p. Thus, as observed in the attached figure, we have:



Simplified system
(Small strains)

L, length considered in the studied direction of the deck.

ΔL, additional deformation produced by the increase in temperature, shrinkage…, in the deck.

Δθ, increase in turn caused by the previous causes.

e, Δe, small magnitude in any position of L (e.g. in the tile-to-tile joint), and its corresponding increase, respectively.

Since we are in the field of the small strains, and taking into account the premises, we may state:

For P3 → $\Delta\theta / \Delta L = M^* / EI$
For P4 → $\Delta L / \Delta e \approx L / e \rightarrow \Delta L \approx L \cdot \Delta e / e$

(Since the deck is the restricting factor)

$$\Delta e \;/\; e \approx EI \;/\; M^* \cdot \Delta\theta \;/\; L$$

(EI, deck stiffness)
(M*, flector moment to be analysed)

By Hooke's Law, for floor tiles,
by imposition of the deck strains:

(Ep, Young's modulus tile)

$$\sigma_p = E_p \cdot \varepsilon$$

where $\varepsilon = \Delta e / e$

Then, considering the previous expressions, this gives:

$$\Delta e \;/\; e \approx EI \;/\; M^* \cdot \Delta\theta \;/\; L$$
$$\sigma_p = Ep \cdot \Delta e \;/\; e$$

(Ep, Young's moldulus tile)

NORMAL STRESS IN THE TILE

$$\boxed{\sigma_p \approx E_p \cdot EI \;/\; M^* \cdot \Delta\theta \;/\; L}$$

$M^* = C \cdot M_{max}$ (C, according to system)

As a complement, the orientational estimation is useful, by simple proportion, of:

$$\boldsymbol{\sigma_{adhesive} = (E_{adhesive} / E_p) \cdot |\sigma_p|}$$
$$\boldsymbol{\sigma_{deck} = (E / Ep) \cdot |\sigma_p|}$$

In the contact area with the tile
(whose sign needs to be analysed)

For a better understanding of its scope we shall apply this to the basic case of the Base Reference [2], as indicated at the end of section 2, while the function Δθ can also be appreciated according to its dependence.

5. RESOLUTION OF A PRACTICAL CASE, VERIFIED, BY MESES

Synthesising Base Reference [2], limiting it to the maximum normal stresses (σ_x, in this reference, obtained with the FEM); the heading responds to a system formed by a rectangular reinforced concrete deck, simply supported by the longer sides, with an adhesive layer and ceramic tiling, taking into account only the temperature effect ΔT= -20º C (i.e. without considering the weight of the system), in accordance with the following characteristics:

Adapted from [2]

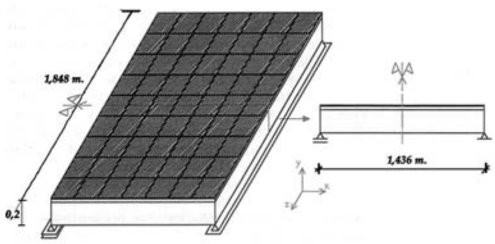
SYSTEM (Affected ΔT = -20°C)	**Young's modulus** E, Gpa	**Thickness, mm.**	$\alpha_L \cdot 10^{-5} \cdot {}^{o}C^{-1}$
Concrete deck	30	200	1
Ceramic tiling, tiles of 200x200 mm^2	70	9	0,5
The bedding, adhesive to be studied: » Thick-bed mortar	20	30	1
» Cementitious based thin-bed	10	6	1

(Only by effect of ΔT) FEM yielded:

σ_{tile} **= -5 MPa**

$\sigma_{adhesive}$ **= 0,4 MPa**

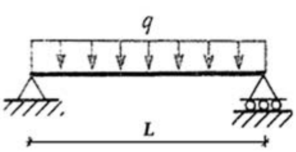
σ_{deck} **= 1,2 MPa**



Solving for MESES, according to the process inferred in sections 3 and 4,

1. Simplified deck.

In regard to the deck, the simplification is a beam 1 m wide in the direction of the span, with its own weight uniformly distributed over itself, i.e. (which MESES does consider).



q

$M_{máx} = qL^2/8$

$I = bh^3/12$

(L = 1.436 m, b = 1 m, h = 0.2 m)

The figure shows its characteristics.

2. Obtaining $\Delta\theta$ max.

As a simply supported beam is involved, with q in all the span, $\Delta\theta$ max. occurs equally at both ends (in x = 0 and x = L). It suffices to determine previously the increase function $\Delta\theta$, by simple partial derivations of θ, obtainable in any handbook [15]

$$\theta = [q / (24EI)] \cdot (L^3 - 6Lx^2 + 4x^3)$$

$$\Delta\theta = [q / (8EI)] \cdot [(L^2 - 2x^2)\ \Delta L + 4x\ (x - L)\ \Delta x]$$

For x = 0, $\Delta x = 0$
(o x = L, $\Delta x = \Delta L$)

$$\downarrow$$

$$\mathbf{\Delta\theta_{max} = (qL^2 / 8EI) \cdot \Delta L}$$

Where ΔL depends on ΔT_{mean}

$$\Delta L = (\alpha_{deck} - \alpha_{tile}) \cdot \Delta T_m \cdot L$$

$$\Delta L = (1 - 0{,}5) \cdot 10^{-5} \cdot (-20) \cdot L$$

(Since the temperature variation, ΔT= - 20 ºC is uniform, therefore $\mathbf{\Delta T_m}$ = - 20 ºC, with the coefficients of thermal expansion α_{deck}= $1\cdot10^{-5}$ ºC^{-1}, α_{tile} = $0.5\cdot10^{-5}$ ºC^{-1}, y siendo $\alpha_{difference} = \alpha_{deck} - \alpha_{tile}$).

3. Determination de σ_P.

Together with 1 and 2, with Young's moduli of the deck and the tile, and M* = M_{max}. (C = 1, in this case, simply supported deck and uniform load),

$$\boldsymbol{\sigma_p \approx E_p \cdot EI / M^* \cdot \Delta\theta / L = E_p \cdot [EI / (q \cdot L^2/8)] \cdot [q \cdot L^2/8EI \cdot (1-0.5) \cdot (-20) \cdot L] / L}$$

simplifying, and operating: $\sigma_p \approx$ **-7 Mpa** (compression)

And complementing, we can estimate the stresses in orientational form, studying their sign...

$\boldsymbol{\sigma_{adhesive} \approx (E_{adhesive} / E_p) \cdot |\sigma_p| = 2}$ **MPa** (Tension)

$\boldsymbol{\sigma_{deck} \approx (E / E_p) \cdot |\sigma_p| = 3}$ **Mpa**

Note: In this case a thick-bed adhesive has been considered, the thin-bed adhesive would give, for the adhesive, stresses half its value (due to its $E_{adhesive}$).

Prior to the conclusions, we need to make the following analysis:

COMPARATIVE ANALYSIS. MESES AND FEM RESULTS		
	Base Reference (FEM)	MESES
σ_{tiles}	-5	-7
$\sigma_{adhesive}$	0,4	2*
σ_{deck}	1,2	3*

** Thick-bed*

The differences are to be considered possibly in relation to

- The effect of deck load, which FEM does not consider (in this reference).
- Overestimation of deck simplification ($\approx$ 17% / $\sigma p \approx$ -5,8 MPa)

General observation: In this case σp could be obtained more directly, given the simplicity of the deck.

6. **CONCLUSIONS.** POSSIBLE FUTURE STUDIES.

It may briefly be noted that a simplified method can complement, and even respond to some questions proper to the technician, with greater simplicity than an advanced method.

Thus, in the case presented, schematically we would have:

<table>
<tr><th>Influence of</th><th>FEM (Base Reference)</th><th>MESES</th></tr>
<tr><td>Deck span</td><td colspan="2">Both respond the same</td></tr>
<tr><td rowspan="2">Thickness and elastic modulus of the deck</td><td>Only intuits</td><td>Responds ($\sigma_{tile} \approx Ep \cdot EI/M^{*} \cdot \Delta\theta/L$)</td></tr>
<tr><td colspan="2">[If thickness or E increases, then σ_{pieza} also increases]</td></tr>
<tr><td rowspan="2">Length of the ceramic tiles</td><td>Responds</td><td>The study needs to be continued</td></tr>
<tr><td colspan="2">[On the level of stresses, not of stability]</td></tr>
</table>

Therefore, MESES:

- Presents a simple method of first estimation/decision
- Enables understanding the conclusions of other models (FEM).
- Responds to certain questions more easily for the technician.

These characteristics are fostered by the possibility of envisaging of future studies, which in this case follow three lines:

1. DATABASE.
Creation and analysis
(of floor deterioration).

- Delimited graphic information. (Advanced topographic techniques).
- Data processing and classification.

2. Implementation of MESES.

- Verification with decks (according to 1).
- Revision of the approach of σp.
- Obtainment of vertical and shear stresses.
- Creation of computer programs.

3. Advanced analysis.

- Need to address in the Project: thermal effects by orientation… etc.
- Symbiosis of MESES in FEM, FDM…

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