

MODELLING THE BEHAVIOUR OF CERAMIC TILE COVERINGS

Miguel Abreu⁽¹⁾, Vítor Leitão⁽²⁾, J. Carvalho Lucas^{†(1)}

(1)Laboratorio Nacional de Engenharia Civil (LNEC), Lisbon, Portugal,

(2)Instituto Superior Técnico (IST), Universidade Técnica de Lisboa (UTL), Lisbon, Portugal

ABSTRACT

The main defects in a direct adhered tile system are cracking and detachment. These defects are due, among other factors, to the lack of knowledge by the designers, and by the absence of guidelines in what concerns the actual behaviour of these types of tilings under service load conditions. This paper presents a recently concluded master thesis work^[1] that attempts to further develop the knowledge in this area by contributing some new finite element solutions of tile systems subjected to characteristic types of loads.

In this type of systems, which are made up of one layer of ceramic material, a tile, glued to a support by a mortar or adhesive, stresses are highly dependent on the difference of the physical and mechanical characteristics of the materials used. In fact, the differences in the thermal and hygrometrical characteristics originate differential strains between the layers, which (when constrained) then cause stresses in the system even in the absence of mechanical loads.

In order to obtain reasonable solutions of characteristic stress fields in tile systems a hybrid-mixed stress finite element formulation for plane states was used^[2]. A cross section of a concrete slab with a ceramic tiling is modelled and analysed assuming linear elastic behaviour. All relevant physical and mechanical material properties were found in bibliographic references^[3 to 9]. The models are subjected to a 0,1 mm/m strain imposition in the ceramic tiles. Results are first obtained for two models, which represent the two main types of tile systems, the so-called thick bed or traditional method and the thin bed or non-traditional method. The difference between these systems is basically due to the type of materials used to glue the tile to the concrete slab, a thick layer of mortar for the traditional and a (comparatively) thin layer of cement-based adhesive for the non-traditional. These two models will be used as reference for a parametric study involving all the relevant geometrical and material model characteristics. The effect of introducing an expansion joint (a perimetral or intermediate one), and other types of loads (three other types of differential strains and a distributed weight) are also studied. Due to the linear elastic material behaviour of the models, the results from these five load types may be used to simulate a wide array of more complex load combinations. Subsequently, in a rough design example, elastic stresses due to a load combination are compared with a failure criterion for the tile/adhesive interface found in the literature reference^[7]. Finally, the importance and limitations of simple linear elastic analyses and design will be discussed. With this in mind, possible future research topics will be presented.

† Deceased on the 25th of October 2003.

1. INTRODUCTION

The detachment and cracking of ceramic tilings, which occur frequently in walls and floors (even in new buildings), have three main causes: application deficiencies, lack of knowledge by the designers, and also by the absence of guidelines concerning the actual behaviour of these types of tilings under the relevant service load conditions. An accurate design of a ceramic tiling solution is only possible once this behaviour is better known. An improper design may result in solutions that are unsafe not only in terms of economic risk, but that may even represent a human safety risk (if tiles fall from tall buildings).

In order to know the behaviour of a ceramic tiling, we need to know its response to the service loads. This response may be expressed in terms of the induced strain and stress fields. There are two main ways of analysing this response: experimentally and numerically. In this work, only numerical analyses are performed using a finite element method.

2. THE DIFFERENTIAL STRAINS CONSTRAINT PROBLEM

Ceramic tilings directly adhered to a support are quite a complex system, namely, the ceramic tiles, the adhesive layer (tile bed), the joints between the tiles and, finally, the support. The support is, usually, for floors a concrete slab, and in walls, a brick wall. In this work, only the floor installation on a concrete slab will be analysed. Because of the direct bond, all components will share the imposed loads, and any differential strain between them will be restrained: these then generate stresses in the tiling. In normal conditions, restraining the differential strains seems to be able to generate larger amounts of stress. For this reason, differential strains are probably the main cause of most of the recorded failures. Therefore, in this study we will focus mainly in the differential strains constraint problem.

The support and the different tiling components, if unrestrained, will suffer different dimensional variations even when subject to the same external action. That occurs because of their different nature, mainly, the different physical characteristics (namely the thermal and moisture expansion coefficients) and mechanical characteristics (specifically the modulus of elasticity). When such systems are directly adhered, these different dimensional variations will be restrained, and stress fields will be generated. The intensity and type of these stress fields will be mostly a result of the amount of strain restrained and the stiffness of the interacting components. The stiffness of the components depends on the geometry and the mechanical characteristics, namely the elasticity modulus (which is a constant in a linear elastic analyses, and establishes the relation between the strain (imposed or restrained) and the stress generated), and the Poisson's ratio (which establishes the relation between the strains in each direction). The amount of strain restrained, results in the amount of differential strain imposed and the relation between the stiffness of the components that are restrained and the components that will restrain that strain.

These stresses will generally consist of quite important compressive or tensile stresses in the direction parallel to the tiling (horizontal stresses – S_{xx}) and related to these stresses variations, compressive or tensile stresses perpendicular to these (vertical stresses – S_{yy}), and shear stresses (S_{xy}) will appear. Because horizontal stresses (S_{xx}) vary mainly in the vicinity of the joints and the tiling borders, in these regions vertical stresses (S_{yy}) and shear stresses (S_{xy}) may reach high values.

3. THE MODEL

The program used in these analyses^[2] is an implementation of a non-conventional hybrid-mixed stress finite element formulation for plane states developed at ICIST (Instituto de Engenharia de Estruturas, Território e Construção) in IST (Instituto Superior Técnico, Universidade Técnica de Lisboa). These types of finite elements formulation not only give us an approximation for the displacements in the domain, like the conventional ones do, but also approximates, simultaneously and independently, the stress fields in the domain and the displacements at the borders. By doing this, they achieve a good approximation, not only for the displacements, but also for the stresses, even in models with very few finite elements (there are only 33 elements in most of the models used in this work like the one shown in figure 2).

The model simulates a rectangular reinforced concrete slab simply supported on two sides. Ceramic tiles are directly bonded to the slab upper surface, by a thick mortar bed (traditional method) or a thin cement-based adhesive bed (non-traditional method). Applying the symmetry simplification at the middle of the slab (see figure 1) it is possible to define a plane where the displacements perpendicular to that plane are zero. Therefore, a bi-dimensional plane state analysis may be performed. In this type of analysis, displacements in two directions (horizontal and vertical) are approximated along with horizontal, vertical and shear stresses.

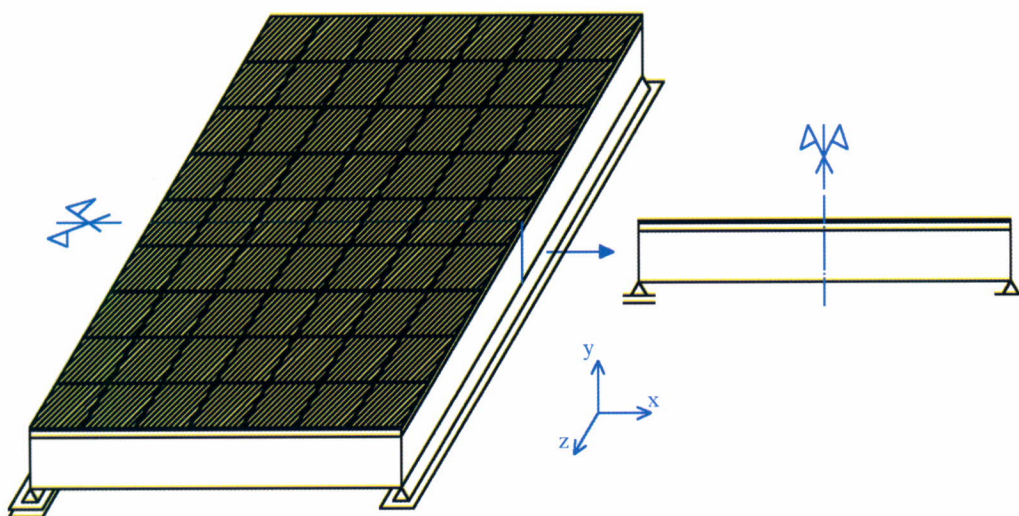


Figure 1 – Modelled slab, bi-dimensional and symmetry simplifications

In the resulting slab section, a symmetry simplification may again be applied, allowing us to model only half of the slab span. If seven tiles are used to cover this span, the modelled section will have only 3.5 tiles (see figure 2). Although such a short slab may not be very realistic, the fact is that, as will be seen in section 6 – Parametric Study, figure 8 d), the behaviour of the model is independent of its length; it is not necessary to simulate a larger span covered with lots of tiles. Therefore, 3.5 tiles are enough to allow us to study the stresses in a tile near a free border and in an interior tile. Stress distribution in a tile near a restrained border will be very similar to the one we may find in an interior tile, except immediately the border. This behaviour will be analysed in section 7 together with the movement/expansion joints. In this section, it is also shown that the stress distribution near the expansion joints is quite similar to the one we may find near the free border.

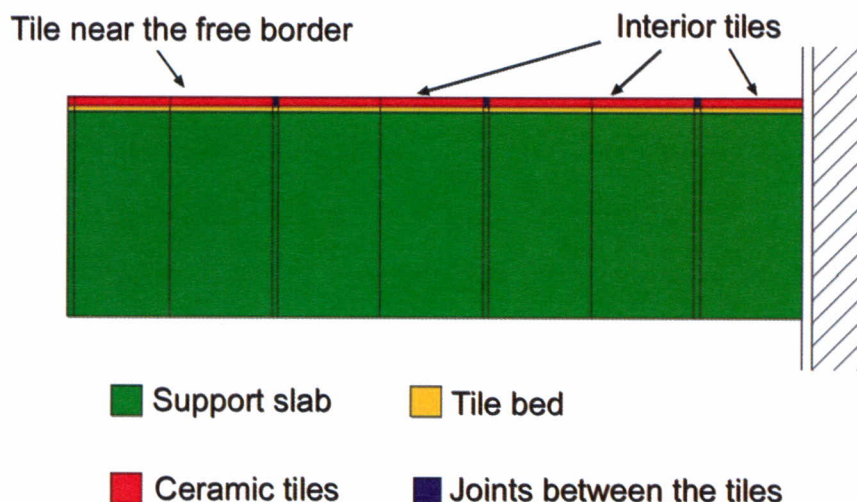


Figure 2 – Thin set reference model: finite element mesh and modelled materials

In order to allow us to analyse the differences of behaviour we may find in a ceramic floor tilling bedded on cement-based adhesives (thin bed, non-traditional method) or bedded on mortar (thick bed, traditional method), two models, representative of each method, will be analysed in sections 4 and 5. In tables 1 and 2, the model geometry and the material properties relevant to a linear elastic analysis (modulus of elasticity and Poisson's ratio) are presented. The only difference between the two models will be the tile bed thickness (being much greater in the thick bed method), and the materials tile bed and joints modulus of elasticity (these are 2 times more rigid in the thick bed method). These two models are called reference models because each will serve as reference to a sensitivity or parametric study, where, one at a time, each of parameter will take the values also presented in these tables. All presented variable values were found in the literature^[3 to 9].

Model geometry (mm)						
Variable	Thin bed (non-traditional) method			Thick bed (traditional) method		
	Reference model	Parametric study		Reference model	Parametric study	
Tile bed thickness	6	2	10	30	15	50
Joint width	6	1	12	6	1	12
Tile thickness	9	6	12	9	6	12
Tile length	200	100	300	200	100	300
Support slab thickness	200	150	250	200	15	250
Support slab span	1436	2672		1436	2672	

Table 1 – Model geometry

	Poisson's ratio (m/m)	Módulus of elasticity (GPa)					
Material	In all models	Thin bed (non-traditional) method			Thick bed (traditional) method		
		Reference model	Parametric study		Reference model	Parametric study	
Tiles	0.30	70	40		70	40	
Tile bed	0.25	10	5	15	20	10	30
Joints	0.25	10	5	15	20	10	30
Support slab	0.20	30	20	10	30	20	40

Table 2 – Model material modulus of elasticity and Poisson's ratio

4. THIN BED (NON-TRADITIONAL) METHOD REFERENCE MODEL

In the present study, the main load acting on the models is a tile 0.1 mm/m expansion (in section 8 other types of loads are also studied). As previously mentioned, different strains between components seems to be, in most of the failure cases, the determinant type of load. As these strains result mainly from the differences in the behaviour of the different materials and being the ceramic tile the “most different” material, large differential strains are expected between the tiles and the other, predominantly cement-based, materials.

The mentioned 0.1 mm/m tile expansion may result from a moisture variation, or from a uniform temperature drop of 20°C (Summer to Winter). When this temperature drop occurs, because the ceramic tile thermal expansion coefficient is approximately $0.5 \times 10^{-5} \text{ K}^{-1}$ they will diminish 0.1 mm/m, but mortars, concrete and the others cement-based materials coefficient about $1.0 \times 10^{-5} \text{ K}^{-1}$, so they will diminish 0.2 mm/m, resulting in a differential strain equivalent to the applied 0.1 mm/m tile expansion. Due to the linearity of the models, resulting stresses will be the same if, instead of applying a 0.1 mm/m tile expansion, we apply a -0.1 mm/m contraction to all other model components: the other components mainly being cement-based material this could have been the result of their drying shrinkage. In addition, also because of this linearity, the presented results may be extrapolated to any other value of differential strain between the tiles and the other components, simply applying the appropriate scale factor.

In the following figure 3, the deformed shape of the thin bed (non-traditional) method reference model when subjected to a tile 0.1 mm/m expansion is displayed (displacements are amplified by a factor of 2300). As shown in this figure, the expansion imposed on the tiles generates the bending of the slab, the compression of the joints between the tiles, and a significant strain of the tile bed near the free border.

In figure 4, the resulting horizontal stress field (S_{xx}) is displayed. As shown in this figure, due to the restraint imposed to the tile expansion, -4.0 to -5.0 MPa compressive stresses are generated in the tiles. The compression of the tiles against each other may generate the well-known phenomena of tile arching. As previously mentioned, if instead of a 0.1 mm/m tile expansion, we have a 0.1 mm/m tile contraction resulting from a uniform temperature raise of 20°C (Winter to Summer),

resultant stresses absolute values will be the same but with inverted sign. So in the tiles 4.0 to 5.0 MPa tensile stresses will be generated. These tensile stresses are high enough to induce significant cracking of the tiling (these types of material expected resistance to tensile stresses is typically only 1.0 to 2.0 MPa). However, such a slow strain imposition will almost certainly permit a significant stress relaxation (mainly at the mortars and cement-based adhesives) inhibiting such high stress values.

Also in figure 4 we can see that, due to the tile expansion, the tile bed and the support slab upper levels are subjected to tensile stresses. Even though the support slab is more distant from the tiles, the tensile stress in the support slab (1.2 to 1.0 MPa) is higher than in the tile bed (0.3 MPa); this happens because the slab is much more rigid and the tile bed is quite thin (6 mm).

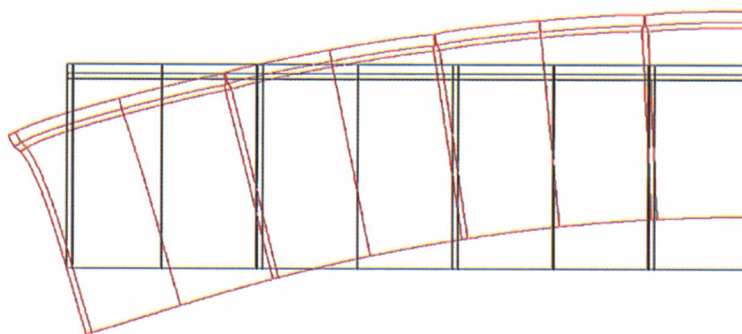


Figure 3 – Model deformed shape (amplified 2300x) due to a 0.1 mm/m tile expansion

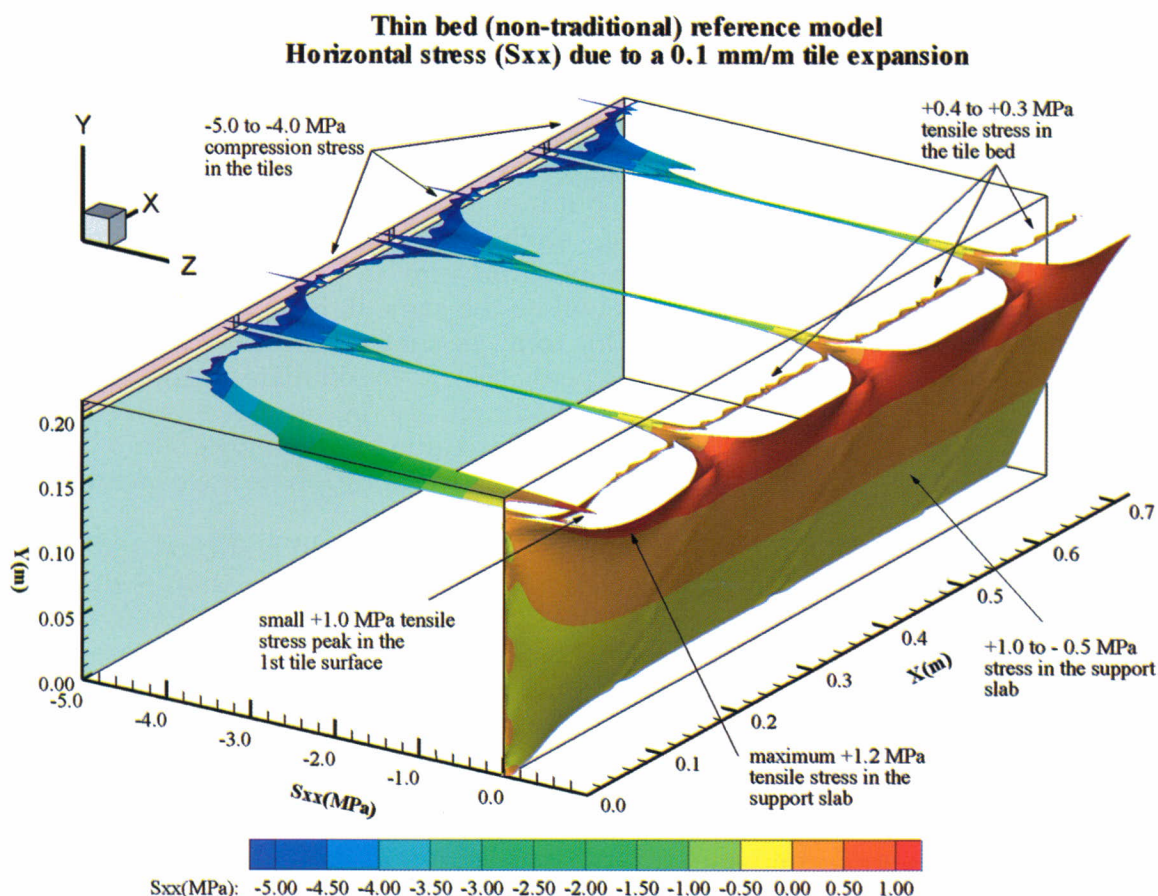


Figure 4 – Thin bed reference model, horizontal stress (S_{xx}) due to a 0.1 mm/m tile expansion

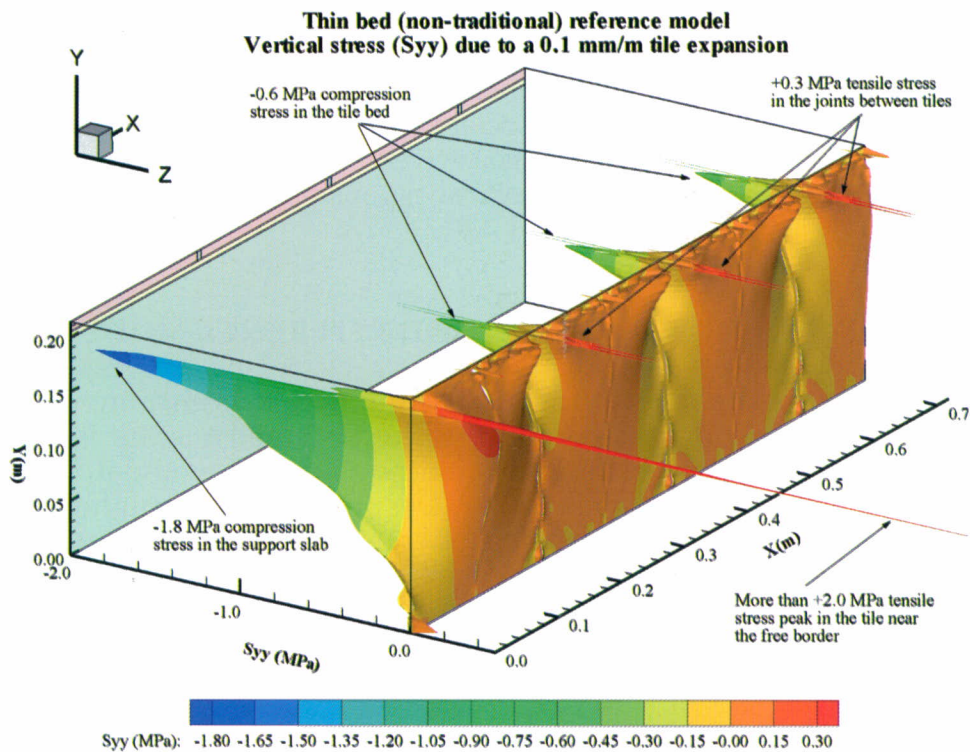


Figure 5 – Thin bed reference model, vertical stress (S_{yy}) due to a 0.1 mm/m tile expansion

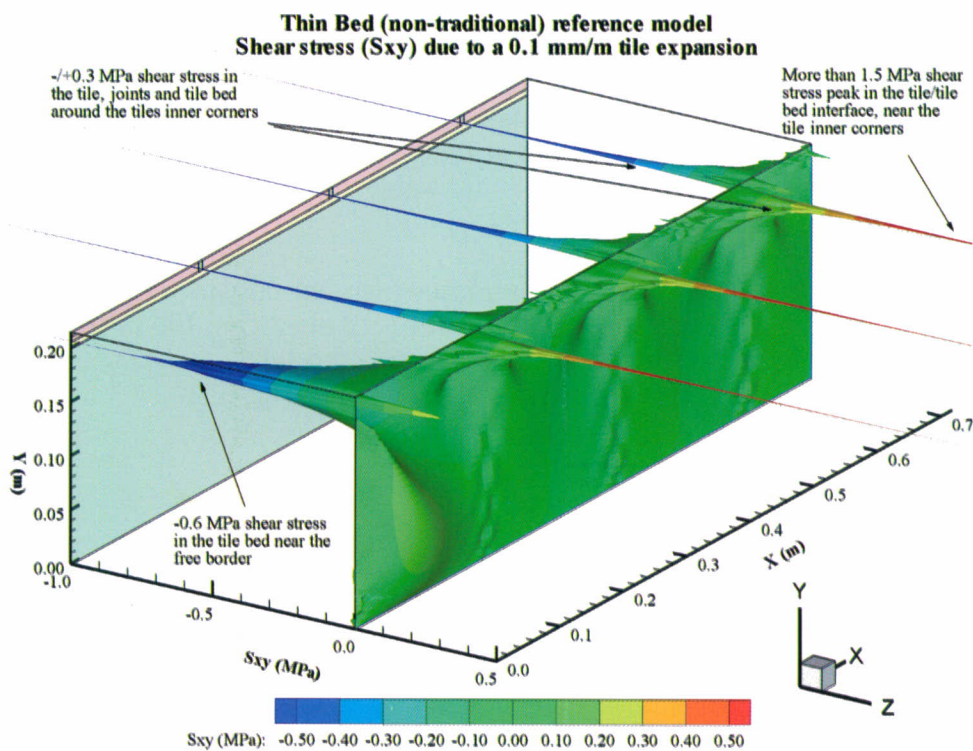


Figure 6 –Thin bed reference model, shear stress (S_{xy}) due to a 0.1 mm/m tile expansion

Near the joints between the tiles and the free border, horizontal stresses are forced to vary significantly, decreasing in intensity or even becoming zero (near the free border). Associated with these horizontal stresses, large variations vertical and

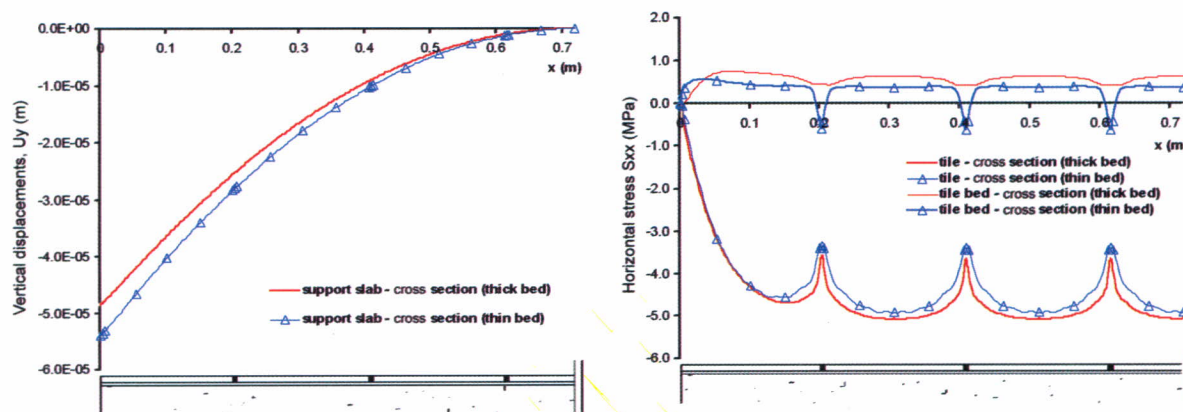
shear stresses will be generated, as shown in figures 5 and 6. Note that the very high but also very thin stress peaks that appear in the vertical and shear stress fields should not be given too much importance as they may be seen to result from the numerical model chosen (the energy associated to these peaks is quite small). With a much smaller intensity, near the joints between the tiles, stresses will also be generated due to the restriction of the vertical expansion of the tiles also imposed by these joints.

5. THICK BED (TRADITIONAL) METHOD REFERENCE MODEL

Applying to the thick bed (traditional) method reference model the same 0.1 mm/m tile expansion, and comparing the obtained results with the ones relative to the thin bed (non-traditional) method reference model, we may find that results are surprisingly quite similar. Despite the much higher stiffness of the tile bed, horizontal stress (S_{xx}) only increases a little (see figure 7b)), and this small increase may result only from the slight increase in the model bending stiffness (revealed by the decrease of the support slab curvature shown in figure 7 a)). Probably the expected larger increase in stresses due to the higher tile bed stiffness is somehow prevented because at the same time the increase in the thickness of this layer helps decrease these stresses. The most likely cause is that the tiles (where the strain is imposed) lie farther apart from the support slab (that is the most rigid element contributing to the imposed strain restriction).

Overall vertical and shear stresses (S_{yy} and S_{xy}), as shown in figures 7 c) and d) are slightly higher in the thin bed reference model, because the decrease in the joint and tile bed modulus of elasticity originates more horizontal stress variation in the joints and near the free border. As discussed in the next section 6 (Parametric Study) when tiles are allowed to deform more, stresses inside the tile tend to decrease but around the tile, the surrounding elements are subjected to an increased strain and so stresses in them tend to increase.

Note that, inside the joints, vertical stresses increase considerably (see figure 7 c) enlarged detail. Due to the increased joint modulus of elasticity, tile vertical expansion restriction is much higher in the thick bed reference model.



a) Vertical displacements

b) Horizontal stress (S_{xx})

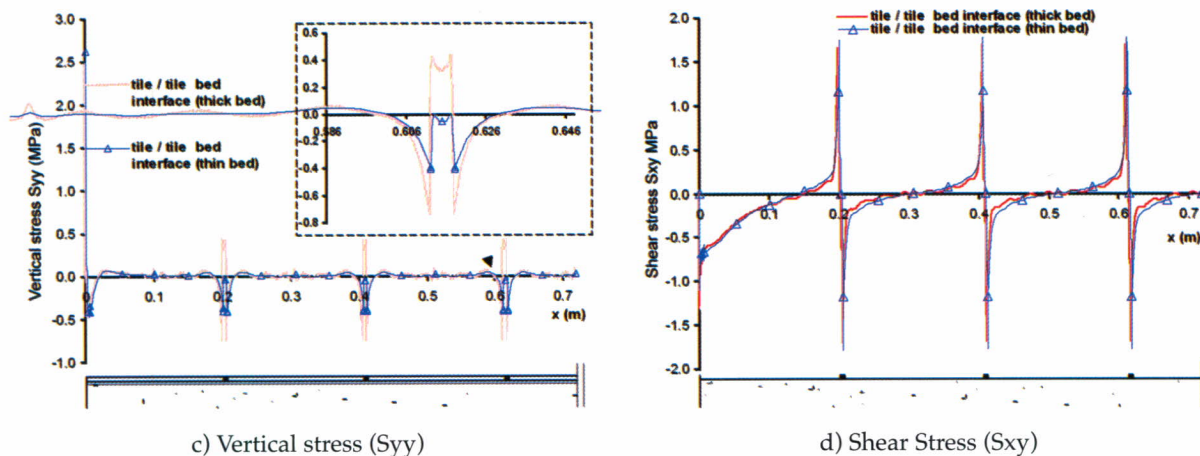


Figure 7 – Thick bed reference model results compared with the thin bed reference model results

6. PARAMETRIC STUDY

The results of the parametric study presented in this section allow us to evaluate the relative importance of each of the model parameters (as shown in table 1 and 2). In general, the thin bed method reference model is slightly more sensitive to all parameter variations.

6.1. TILE BED THICKNESS

Somewhat unexpectedly, even in the thin bed method reference model (the one more sensitive to parameter variations), the changes made in the model tile bed thickness do not cause significant variations in the resulting stresses. A decrease of horizontal stresses in the tile is expected when the thickness of the tile bed increases, because the tile becomes farther apart from the much more rigid support slab compared with the tile bed. As shown in figure 8 a) there is a decrease in these stresses but it is somehow smaller than expected. One reason for this behaviour may be that the decrease expected in these stresses when the tile bed thickness increase is to some extent opposed by the increase of the overall bending stiffness of the support/tiling conjunct. The (assumed) simply supported condition of the slab, and its small span, may result in models being very sensitive to even small bending stiffness variations.

6.2. TILE BED MODULUS OF ELASTICITY

Similarly to the changes in the tile bed thickness, the changes made in the tile bed modulus of elasticity produces only a small change in the model behaviour, even though the reduction of this modulus will originate a slight decrease in the stresses inside the tile and a slight increase of the stresses in the tile bed and joint between tiles (the joint becomes more squeezed). Contrary to the usual situation, in this analysis the thick bed reference model is more sensitive to the parameter variation. This occurs because of the much larger quantity of material being subject to this parameter change in the thick tile bed.

6.3. JOINT WIDTH AND MODULUS OF ELASTICITY

The changes made in the joints (between the tiles) width and modulus of elasticity have an important influence in the model behaviour, but only in the area immediately around these joints. The dissipation of the horizontal stresses that occurs

inside the joints will increase when the modulus of elasticity decreases, or the width increases (see figure 8b)). Because of the increase in the horizontal stress variation, the vertical and shear stresses, mainly in the tile bed, will also increase accordingly.

6.4. TILE THICKNESS

The increase of the tile thickness will make the tiles less susceptible to restrictions to the imposed strain. That originates fewer stresses inside the tiles, but because these are allowed to deform more, in the surrounding elements strain and stresses will increase (see figure 8c)).

6.5. TILE LENGTH

The decrease of the tile length causes a slight reduction of the generated stresses. The models are much more sensitive to large tile length reductions. If the tiles are short enough and the joints between the tiles large and flexible enough, horizontal stresses at the tiles will be limited but again associated with the bigger variation of these stresses, vertical and shear stresses will increase.

6.6. TILE MODULUS OF ELASTICITY

Amongst all parameters, the tile modulus of elasticity is the one with the greatest influence in model behaviour (see figure 8d)). Decreasing this modulus contribute to the overall decrease of all generated stresses. This behaviour occurs because the differential strain is imposed in the tiles.

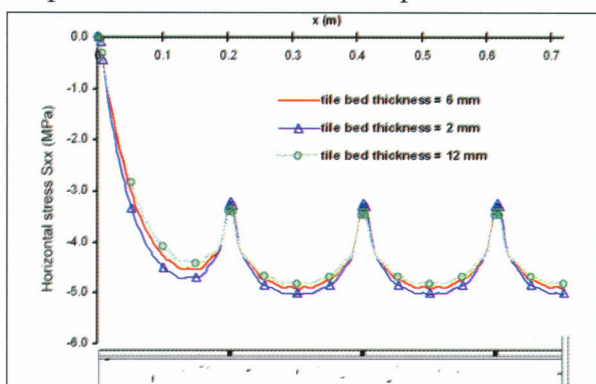
6.7. SUPPORT SLAB THICKNESS AND MODULUS OF ELASTICITY

Increasing the support slab thickness or its modulus of elasticity will lead to higher stresses in the model (see figure 8 e)). This behaviour seems to be fundamentally related with the already mentioned, somehow high influence of the model bending stiffness in the generated stresses.

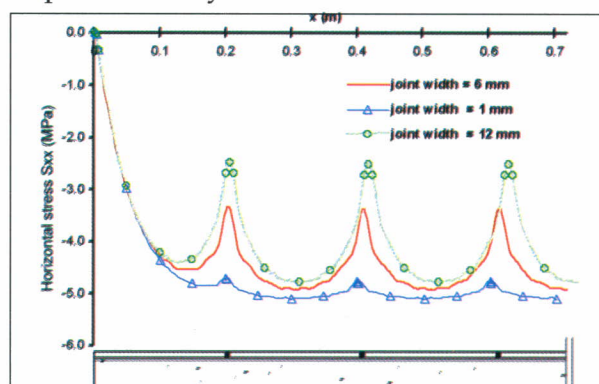
6.8. SUPPORT SLAB SPAN

In these conditions, the behaviour of the models is independent of the support slab span (model length) and number of modelled tiles (see figure 8 f)). This behaviour allows us to limit the model to short spans and few tiles and also explains why the models are quite insensitive to the introduction of intermediate expansion joints. This is due to the fact that stresses depend on strains and these (unlike displacements) are dimensionless.

In conclusion, it is important to state that it is very difficult to analyse the different parameter influence without well-defined failure conditions. The importance of the different parameter will depend mainly on these conditions.



a) Tile bed thickness variation



b) Joint width variation

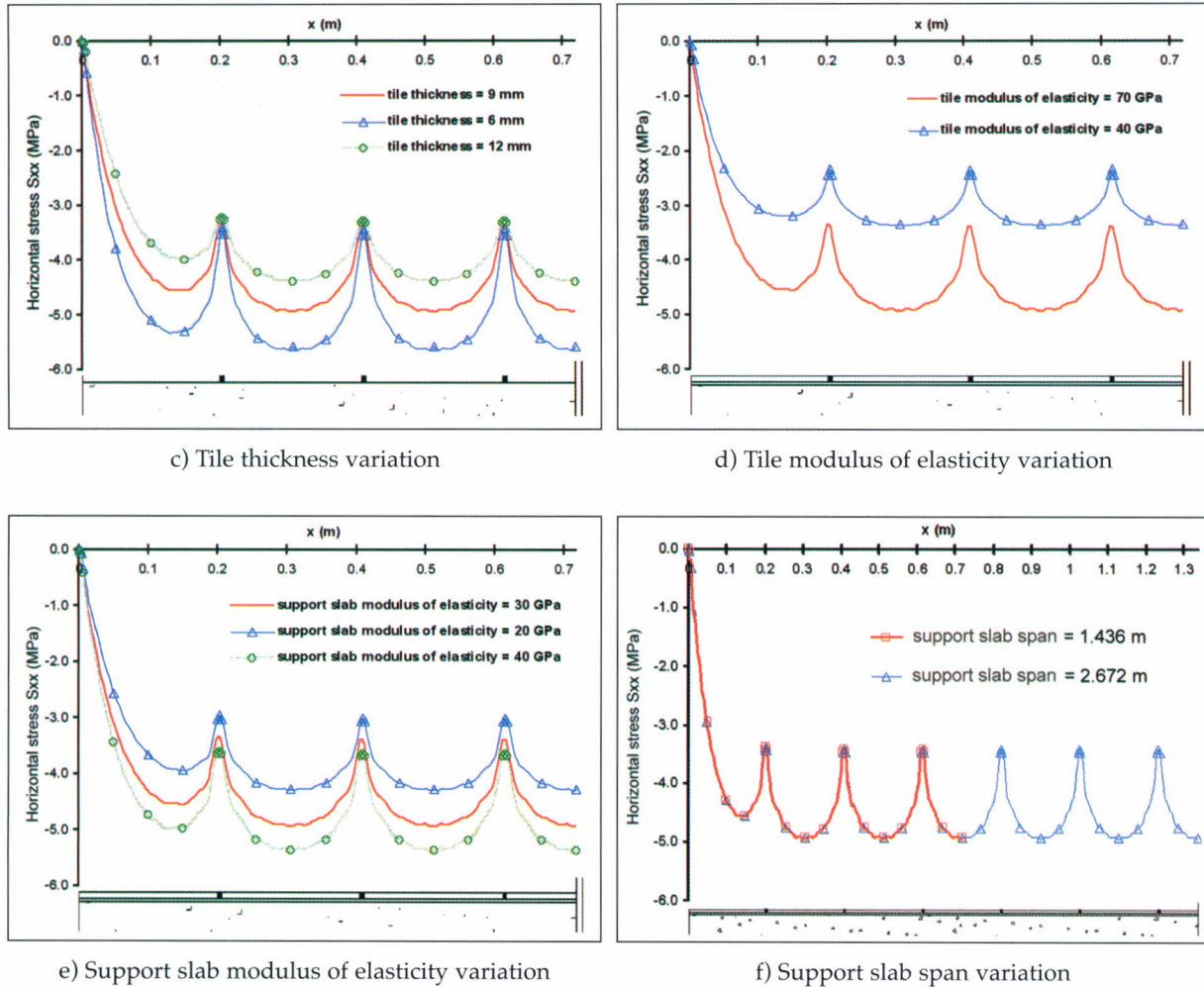


Figure 8 – Parametric study thin bed (non-traditional) model examples, horizontal stress (S_{xx}) due to a 0.1 mm/m tile expansion, along the tile cross section

7. MOVEMENT JOINTS

In this section, the influence of introducing movement joints in the ceramic tiling will be analysed. There are two main types of movement or expansion joints, the perimetral ones (which are applied along the tiling borders) and the intermediate ones (which divide large tiling panels in smaller ones). These types of joints are sometimes recommended in order to limit the stresses that may build up in a ceramic tiling due to differential strain restriction. However, their utilization in directly adhered ceramic tilings is somehow controversial.

7.1. INTERMEDIATE EXPANSION JOINTS

In these model conditions, the introduction of an intermediate expansion joint does not modify the model behaviour, except in the proximity of this joint. As shown in figure 9, in the proximity of the joint, the model behaviour becomes very similar to the one we may find near the free border. This behaviour results directly from the already mentioned model length independence. Therefore, in these conditions, the introduction of these types of joints seems to be unable to reduce stresses far from the joint, and very high vertical and shear stresses are generated near the introduced

expansion joint; these stresses may be prejudicial. Even so, these intermediate expansion joints may prevent the formation of large panels of tiles arching out; the application in lines where, for some reason, cracking of the support may be expected (for example borders between different supports) is essential.

7.2. PERIMETER EXPANSION JOINTS

Because the introduction of perimeter expansion joints only makes sense near a restricted border, in order to study its influence new support conditions were applied to the models. As mentioned before, in a tile near a restricted border stress distribution will become similar to the one we may find in an interior tile, except immediately over the border where stresses will vary greatly and may reach significant values. When a perimetral expansion joint is introduced, the stress distribution near this joint becomes similar to the one we may find near a free border. Far from the joint, the stress distribution does not vary significantly. Even if this means that high vertical and shear stresses will appear, because the already mentioned stresses immediately over the restricted border will probably crack any rigid joint, the execution of perimeter expansion joints is always advisable.

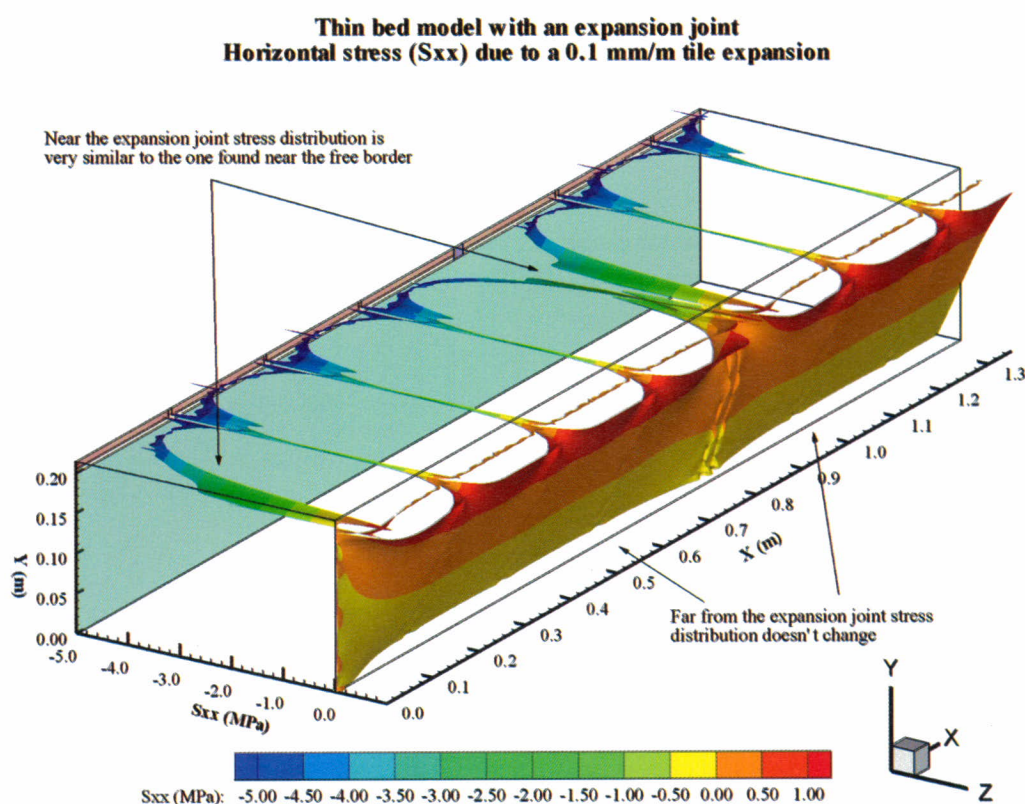


Figure 9 – Thin bed model with an intermediate expansion joint, horizontal stress (S_{xx}) due to a 0.1 mm/m tile expansion

8. OTHER LOADS

So far, the load applied to all models is a 0.1 mm/m tile expansion; in this section, other types of load conditions are also studied. These other types of load conditions are a 0.1 mm/m contraction of the tile bed, the joints between the tiles and the support slab; a uniform 1.0 kN/m load over the tiling surface is also studied.

As shown in figures 10 a), b) and c) the combined effect of the contraction in the tile bed, joints and support slab equals the already analysed tile expansion. However, the separate analyses of these loads allow us to better understand the stresses build up mechanisms. It also allows us to carry out any load combination of different strains in the different materials.

As shown in figure 10 d) the stresses due to the 1.0 kN/m load are quite similar to the ones generated by the tiles expansion, but they increase with the slab span (being zero near the border and maximum near the mid span) and are much lower than the ones due to the tile expansion. Even if a heavier load or larger span is considered, stresses will still be much lower. However, it is important to mention that the stresses due to this load will not relax (decrease over time) and the bending of the slab due to this load will increase over time (due to a creep effect) working as another type of strain imposition.

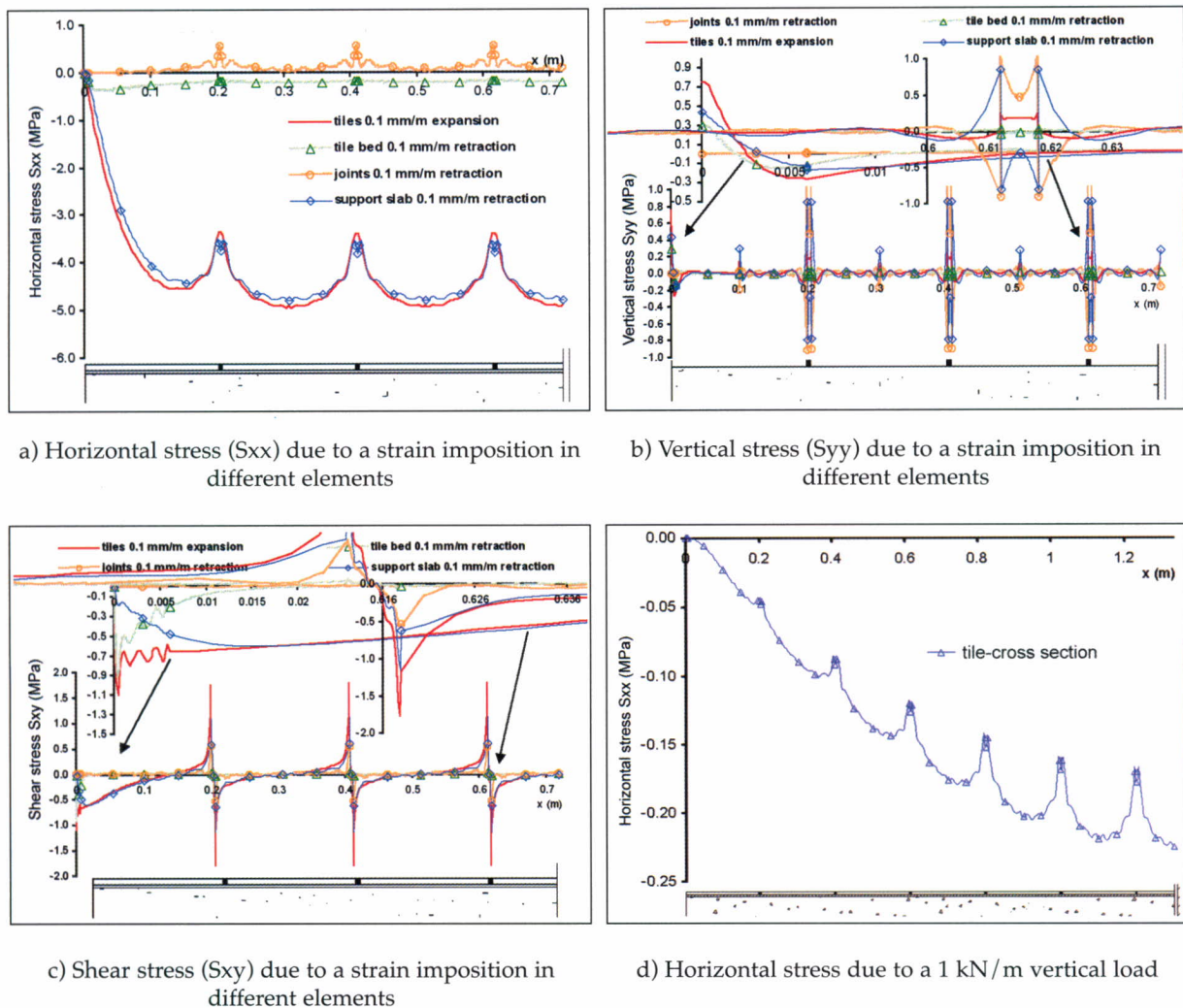


Figure 10 – Stress fields due to various actions, thin bed (non-traditional) reference model

9. DESIGN EXAMPLE

In this design example, stresses in the tile/tile bed interface due to a load combination are compared with a failure condition for this interface as published in a reference^[7]. The main load in this combination is a uniform temperature drop of 20°C; as mentioned before this load is equivalent to a 0.1 mm/m tile expansion. The other

loads are presented in table 3. In this table the load combining factors (serviceability limit state, frequent combination) and relaxation factors are also presented. Without a more precise knowledge of the stress relaxation capability, the usually conservative value of 50% typically used in concrete was extrapolated to the other materials.

SERVICEABILITY LIMIT STATE FREQUENT COMBINATION		Characteristic value		Combination factor	Relaxation factor
Non-variable loads	Tile expansion	0.10	mm/m	1.0	50%
	Tile bed contraction	-0.30	mm/m	1.0	50%
	Contraction of tile-to-tile joint	-0.60	mm/m	1.0	50%
	Support concrete slab contraction	-0.24	mm/m	1.0	50%
	Tiling dead load	0.34	kN/m ²	1.0	*
Variables loads	Main load - Uniform temperature 20°C drop (equivalent tile expansion)	0.10	mm/m	0.5	50%
	Utilization live load	2.0	kN/m ²	0.2	*

Table 3 – Load combination example

In figure 11, vertical and shear stresses along the interface between the first tile near the free border and the tile bed are compared with the mentioned failure condition. This is a Mohr-Coulomb type failure condition, so rupture occurs if the shear/vertical stress pair is outside the represented triangle. As shown in this figure there are three zones where stresses are outside the triangle. However, in zone 1 (near the free border) and zone 2 (near the joint between the tile) this happens along a very short interface length, which may not be enough to originate the detachment (in order for failure occur not only is stress intensity important, but also the size of the zone where stresses are high). On the other hand, in zone 3 (this zone starts 6 mm far from the free border), not only the stresses are high but they also remain high enough along about $\frac{1}{3}$ of the tile surface. So, in this zone detachment may be considered likely to occur.

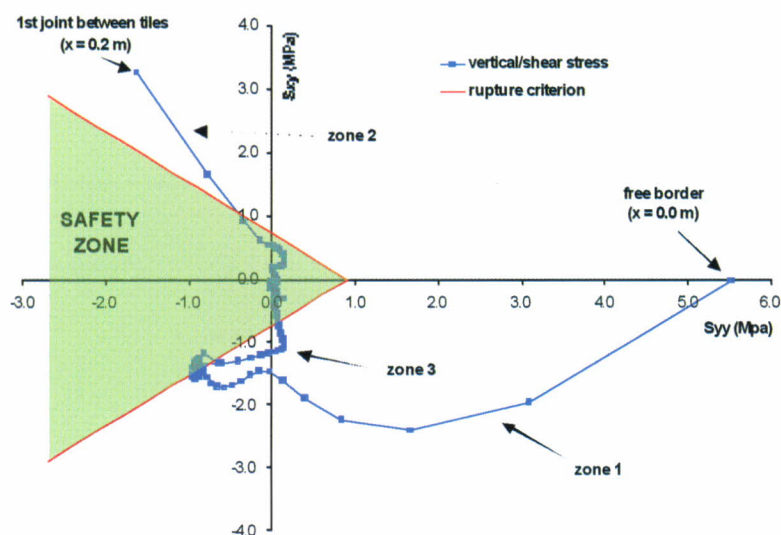


Figure 11 – Shear and vertical stress along the first tile/tile bed interface due to a load combination example compared with a failure condition

10. CONCLUSIONS & FUTURE DEVELOPMENTS

Linear elastic analysis provides us with a fast and simple way to obtain a first approximation of the type and intensity of stresses that build up inside a ceramic tiling. The presented stress distributions match up with some common failure symptoms. Horizontal compression or tensile stresses may cause the tile to arch or crack. High vertical and shear stresses near the tile borders next to joints or free borders may cause the progressive detachment of the tiles located near these borders.

However, the simplifications associated with these simple models, the lack of more exact data about some of the materials properties, some of the actions, and failure conditions, and more importantly, the lack of validation of the numerical results with experimental models, are reasons enough to be careful in applying these models for actual design. For that reason it is very important to proceed with further investigation in this area.

The modelling of the behaviour of ceramic tilings directly attached to the support is essential in order to allow us to design an efficient tiling solution. Without this design, it is not possible to guarantee the solution's economic and safety viability. Only this design will allow us to establish in each condition, important parameters like the geometry solution and the required materials resistance and flexibility. It will also allow us to precisely establish when the directly adhered solution becomes non-viable, and alternatives, like mechanical fastenings, indirectly adhered, or completely non-adhered solutions should be pursued.

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