METHODOLOGY FOR THE DESIGN OF CERAMIC TILINGS BASED ON STRUCTURAL ANALYSIS OF HISTORICAL ARCHIVES

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ABSTRACT

The present communication is set in the frame of a research project entitled "Graphic tools for the cataloguing and design of ceramic tilings and fabrics"[1]*, in which it is sought to develop a Design Pattern Information System (SIMOD) for the textile and ceramic industries.*

SIMOD[1] *is a system for design analysis, resulting data management and creation of new designs from the data. A modular architecture has been defined with functionally different modules that are interconnected. In one of those modules, denominated SAEMOD: Design Pattern Analysis and Editing System, a set of tools has been developed whose objective is the application of the scientific theory of plane symmetry groups, following their classic references, to the cataloguing and design of ceramic tilings.*

In the analysis process, a gradual approximation is made, in which the processed information progressively increases in complexity, from an image formed by pixels, via objects and clusters of objects, to the fundamental parallelogram and plane symmetry group.

1 This project has been supported by the Ministry of Education and Science, in the technological Programme for Research and Development (Project FEDER-CICYT: 1FD97-0402) and by the companies: TAU CERÁMICA, TAULELL S.A. and RAFAEL CATALÁ, S.A.

The importance of these results in the ceramic tiling design process lies mainly in the excellent source of information that they provide. Moreover, understanding and visualising the compositional structure of the analysed tiling, especially if it is complex, invite the designer to reflect on its generative possibilities, driving his creativity from the first design phases.

In this communication, examples are also given of the proposed design methodology based on the cataloguing results. These are fundamentally the outcome of the possibilities that symmetry group theory affords. This enables multiplying the resources available to the designer, offering him the necessary tools for designing or redesigning capriciously entwined complex pieces, automatically visualising the various compositions that can be achieved with them.

1. INTRODUCTION

The methodology developed directs its efforts to making practical use of the scientific symmetry group theory, for the systematic cataloguing and designing of ceramic tilings. For the elaboration of this tool the classic references of symmetry group theory have been reviewed: Rose and Stafford (mentioned in[2]), [3][4][5][6][7][8][9][10]. Symmetry group theory allows classifying ceramic tilings by considering non-concrete cases, hence not belonging to a certain real casuistry, but responding to their theoretical possibilities. It therefore constitutes a different classification from the one made traditionally from a history perspective (e.g.[11][12]) where actual, concrete cases are dealt with.

The interest in symmetry group theory resides in two facts:

1. All the shapes can be classified in terms of the symmetry group to which they belong. Three types of symmetry groups can be distinguished.

    - Point symmetry groups (gsp), which allow classifying isolated shapes.
    - Frieze symmetry groups (gsf), which allow classifying shapes that repeat themselves in a certain direction.
    - Plane symmetry groups (GSP), which allow classifying shapes that repeat themselves in two directions that are not parallel or opposing.

2. The number of symmetry groups is limited, as follows:

    - The gsp can be cyclical (Cn) or dihedral (Dn). The cyclical groups only contain direct isometries (rotations). The dihedral groups combine direct and opposing isometries (reflections).
    - There can only be 7 types of gsf, called according to the international symbol of the group used by crystallography: P111, P1M1, PM11, P1A1, P112, PMA2 and PMM2.

- There can only be 17 types of GSP, called according to the international symbol of the group used by crystallography: P1, PG, PM, CM, P2, PGG, PMG, PMM, CMM, P3, P31M, P3M1, P4, P4M, P4G, P6 and P6M.

To be able to apply the symmetry group concept to the field of ceramic tilings, a ceramic tiling are considered to have three formal components: the tile that constitutes the primary unit of the tiling; the printed motif on the tile, which represents the particular characterisation of a tile; and the repetition rule, as a peculiar arrangement of the tiles on the tiling background. Each of these components can be classified according to the symmetry group to which it belongs, thus the tile and the motif, as isolated shapes, will belong to a certain gsp and the repetition rule, as a regularly repeated set of shapes, will belong to a certain GSP.

gsp	number of aspects	GSP	Type of PF	Associated isohedral tilings	Periodic patterns	Associated point patterns
C1	1D	P1	P-RE-RO-C-ROE	IH1,41	PP1	DPP8,16,20,41,51
	2D	P2	P-RE-RO-C-ROE	IH4,23,46,47,84	PP7	DPP7,8,13,15,16,19,20,41,50
	3D	P3	ROE	IH7,33	PP21	DPP21,25,28,49
	4D	P4	C	IH28,55,79	PP30	DPP30,35,38,39,41
	6D	P6	ROE	IH21,31,39,88	PP42	DPP42,47,48a,48b,50
	1D1R	PG	RE-C	IH2,3,43,44	PP2	DPP13,20,51
		PM	RE-C	IH42	PP3	DPP15,16,41
		CM	RO-C-ROE	IH22,45,83	PP5	DPP16,19,41,50
	2D2R	PGG	RE-C	IH5,6,25,27,51,52,53,86	PP9	DPP9,13,16,19,20,41,50,51
		PMG	RE-C	IH9,59	PP10	DPP11,15,19,41,50
		PMM	RE-C	IH48*	PP14	DPP14,15,16,39,41
		CMM	C-RO	IH54,78	PP17	DPP15,16,17,37,38,41
	3D3R	P31M	ROE	IH30,38	PP23	DPP23,48a,48b
		P3M1	ROE	IH87*	PP27	DPP27,47,50
	4D4R	P4G	C	IH56,81	PP33	DPP33,38,39,41
		P4M	RE-C	IH80*	PP37	DPP37,38
	6D6R	P6M	ROE	IH77	PP46	DPP46
C2	1D	P2	P-RE-RO-C-ROE	IH8,57	PP8	DPP8,16,20,41,51
	2D	P4	C	IH61	PP31	DPP31
	3D	P6	ROE	IH34	PP43	DPP49
	1D1R	PGG	RE-C	IH9,59	PP10	DPP20,41,51
		PMG	RE-C	IH58	PP12	DPP16,41
		CMM	RO-C-ROE	IH60*	PP18	DPP16,41
C3	1D	P3	ROE	IH10	PP22	DPP51
	2D	P6	ROE	IH90	PP44	DPP50
	1D1R	P31M	ROE	IH89*	PP24	DPP50
C4	1D	P4	C	IH62	PP32	DPP41
	1D1R	P4G	C	IH63*	PP34	DPP41
C6	1D	P6	ROE	IH11	PP45	DPP45

Table 1. Cataloguing of a ceramic tiling with tiles or motifs with a cyclical symmetry group.

The method proposed to classify ceramic tilings is based on the classifications of[5] for periodic patterns (PP), point patterns (DPP) and isohedral tilings (IH). The tiling pattern that covers the plane by repeating the directions marked by its sides has also been considered in terms of the fundamental parallelogram (PF). Of[8] the following types of PF have been considered: oblique parallelogram (P), rectangle (RE), square (C), any rhombus (RO) and rhombus formed by two equilateral triangles (ROE).

The tables for the adopted cataloguing (Table 1 and Table 2) have been found by unifying and rearranging the classifications of[5], adding consideration of the type of PF. In these tables, knowing the type of tile gsp or motif, the number and type (direct or reflected) of aspects with which the tile or motif is shown in the design and the type of PF, enables determining to which type of GSP the analysed tiling belongs.

gsp	number of aspects	GSP	Type of PF	Associated isohedral tilings	Periodic patterns	Associated point patterns
D1	1	PM	RE-C	IH64	PP4	DPP16,41
		CM	RO-C-ROE	IH12,14,68	PP6	DPP20,41,51
	2	PMG	RE-C	IH13,15,66,69	PP13	DPP13,20,41,51
		PMM	RE-C	IH72	PP16	DPP15,41
		CMM	RO-C-ROE	IH26,67,91	PP19	DPP16,19,41,50
	3	P31M	ROE	IH16,36	PP25	DPP25,49,51
		P3M1	ROE	IH19*	PP29	DPP28,49
	4	P4G	C	IH29,71	PP35	DPP35,41
		P4M	C	IH70*	PP39	DPP39,41
				IH82	PP38	DPP38
	6	P6M	ROE	IH92* IH32,40	PPP47 PP48	DPP47,50 DPP48a,48b
D2	1	PMM	RE-C	IH72	PP6	DPP16,41
		CMM	RO-C-ROE	IH17,74	PP20	DPP20,41,51
	2	P4G	C	IH73	PP36	DPP41
		P4M	C	IH76	PP41	DPP41
	3	P6M	ROE	IH37	PP49	DPP49
D3	1	P31M	ROE	IH18	PP26	DPP51
		P3M1	ROE	IH19*	PP29	DPP51
	2	P6M	ROE	IH93	PP50	DPP50
D4	1	P4M	C	IH76	PP41	DPP41
D6	1	P6M	ROE	IH20	PP51	DPP51

Table 2. Cataloguing of a ceramic tiling with tiles or motifs with a dihedral symmetry group.

2. ANALYSIS METHODOLOGY

The Analysis tool executes a set of Operators that act on the input data (bit maps or vectorial image), and generates new output data. The initial data come from the Acquisition database and the final ones, are fed into the Design database.

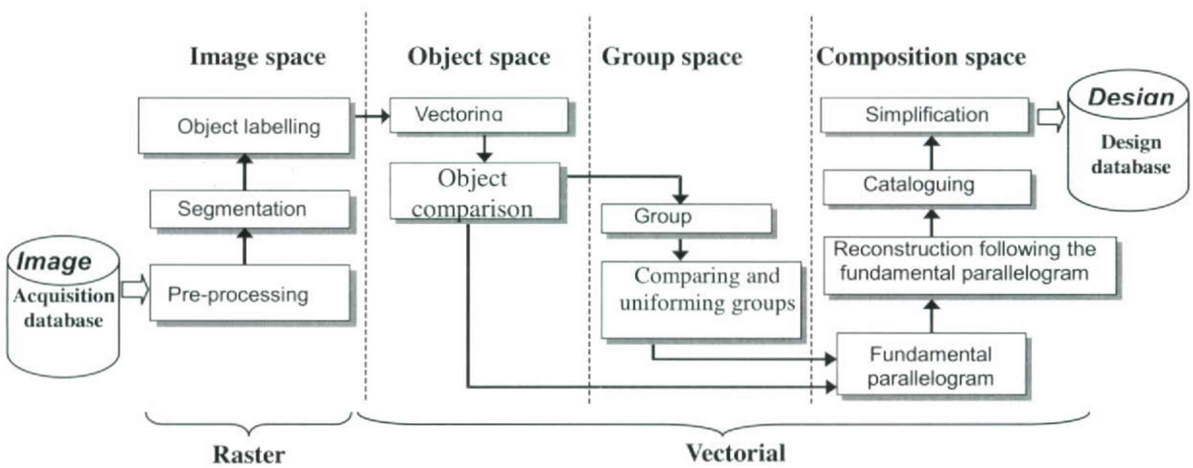


Figure 1: Work spaces and analysis operators.

Figure 1 shows the organisation of the operators that make up the analysis tool, as well as their classification in various stages according to the complexity of the data involved (representation space used):

- Image space: the pixels of an image are used, employing image processing, segmentation and labelling techniques, until finding the objects that make up the image.
- Object space: the objects made up of pixels are converted to groups of objects defined by vectorial boundaries, which are compared and whose symmetry axes established.
- Group space: these objects are grouped, also comparing the groups of objects, and the object and group symmetry axes are found.
- Composition space: based on the comparisons made at object and group level, the repetitions and symmetry axes existing in the design are analysed, determining the fundamental parallelogram and plane symmetry group.

Figure 2 depicts the image of a ceramic motif. This motif will be used as an example in the following points to show the different workspaces and operators.

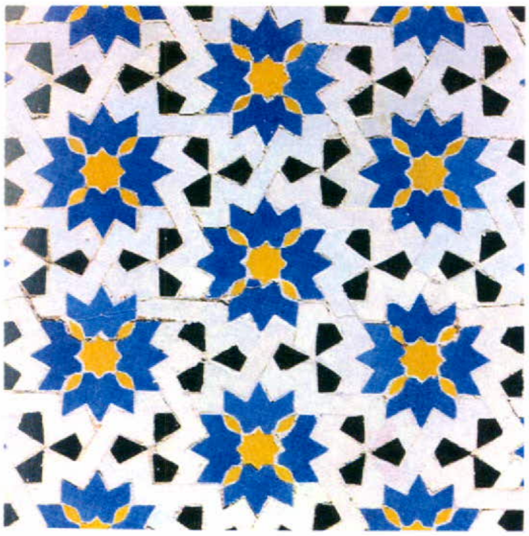


Figure 2: Image example of a historical decorative motif.

2.1 IMAGE SPACE

In the image space, bit map images are used (Figure 3). On being the first part of the analysis process, the input is made up of the images from the Acquisition database. The output continues to be an image but each region (object) is conveniently labelled with an index (value of each of its pixels), which distinguishes it from the rest of the regions and from the background.

There are various operators[13], although a similar routine is always followed:

- Smoothing: elimination of noise.
- Segmentation[14][15][16]: decomposition of the image in regions differentiated by some attribute, such as colour or texture.
- Labelling: assignment of an index to each of the differentiated regions.

2.2 OBJECT SPACE

Taking as input the labelled image, a vectorial data structure is generated consisting of a list of objects - which will be the output information of the object space - each of which is made up of a set of properties (colour, area, etc.) and a list of boundaries (one external one and any number - including zero – of internal boundaries), which delimit the object region. The boundaries are formed by an alternate sequence of nodes (points) and segments (straight, circumference arches and cubic curves) arranged in a cyclical form (the last segment is followed by the first node).

Figure 4 shows the boundaries of the objects found in the fragment of the image example of Figure 2. In the selected object (framed in blue) the succession of nodes and segments can be observed.

Inside this workspace, two clearly differentiated phases exist, which correspond to two operators:

- Vectoring[17][18]: this approximates the boundaries [19] that delimit the object by a set up of segments (straight, circumference arches and cubic curves[20]). This representation is more manageable and compact.
- Object comparison[21]: a comparison is made on two levels, in the first place simple parameters (area, perimeter, etc.), and if they are very similar, detailed comparison is made of their shape by the signature[19] of the object (functions, module and angle of the radius vector that goes from object mass centre to the external boundary). This comparison provides the transformation that relates two identical objects, expressed with a reflection regarding the horizontal axis (if necessary), a rotation and a shift. Comparing an object with itself yields the circular or mirror symmetry axes that it may have.

Both the comparison data between the objects and the symmetry axes are kept in each object, enriching the data structure.

Figure 3: Detail of the image in Figure 2 (Image space).

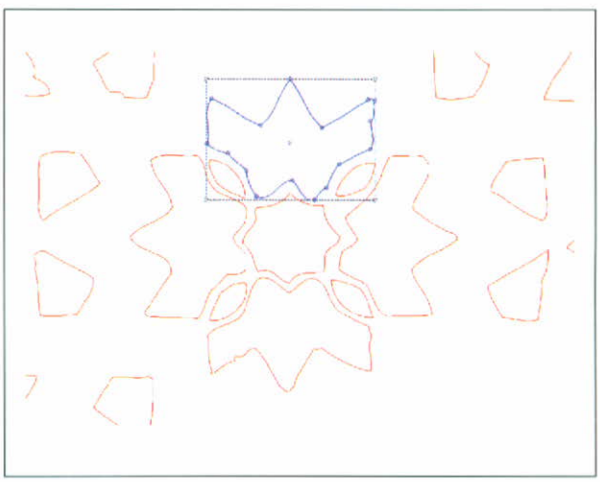


Figure 4: Result of vectoring the image in Figure 3 (Object space).

2.3 GROUP SPACE

Taking the vectorial data structure with the list of objects as input, we will generate a list of groups, in which each group is made up of a group of related objects by means of perceptual criteria and a boundary (minimum convex polygon that includes these objects). Now the output will consist of two lists, the list of objects and the list of groups.

Figure 5 depicts the objects belonging to the same group, surrounded by a boundary, in the same colour. As grouping criterion only contacts have been used.

This workspace is similar to that of objects, since work entities (objects or groups) are found first and they are compared later, even though the methods used are quite different:

- Grouping[22][23][24][25]: a process is used that groups together objects related to each other by means of perceptual criteria (inclusion, contact, co-circularity, co-linearity, symmetry and overlapping).
- Comparing and uniforming groups: comparison is carried out at two levels. In the first place it is established that the groups contain a certain number of identical objects, and subsequently the transformations are compared (shifts, rotations or symmetries with slipping), which relate the same objects between the two groups. The existence of a predominant transformation indicates that the groups are made up of the same objects and that these are distributed in an equal way inside the group to which they belong. These groups are considered equal and have been uniformed. Uniforming is performed by substituting objects in groups by other objects that occupy the same position inside another group, so that all the groups are made up of exactly the same objects in the same arrangement.

As with the objects, the comparison data between groups and their symmetry axes are stored in each group of the list.

2.4 COMPOSITION SPACE

Analysing the vectorial data structure with the lists of objects and groups, information is generated on the structure of the design: the two vectors that define the sides of the fundamental parallelogram and the plane symmetry group (symmetry axes and existing rotation centres)[5][7]. This structural information, together with the lists of objects and groups comprises the Analysis output information.

Figure 6 depicts the identical groups in the same colour, and their symmetry axes have been drawn. The fundamental parallelogram appears in black, an equilateral rhombus having been detected. Only the groups remain that are contained in the fundamental parallelogram, however without dividing any group. The yellow group appears with two different orientations. The resulting cataloguing is CMM considering the motif with four symmetry axes, since this is more restrictive than if we consider the motif with two orientations and three axes (P6M).

The operators of the composition space are as follows:

- Fundamental parallelogram: starting from objects or identical groups, in which the transformation involved is only a shift, we seek two unique vectors that allow us to move between them by means of linear combinations (these two vectors act as a base for the vectorial space defined by the positions of identical elements). When there are several different bases, we choose the one that has a multiple area, since the repeatability of the design is that of the least repeated elements. These vectors form two sides of the fundamental parallelogram, i.e., the minimum part of the design that by replication and displacement is able to generate it all.

- Reconstruction: the fundamental parallelogram represents repeatability, which allows us to establish if all the groups that overlap, shifting in the directions indicated by the two vectors of the parallelogram, are identical. If they are not, we will eliminate some and we will replicate others until the design has been homogenised.

- Cataloguing: considering the geometry of the fundamental parallelogram, the symmetry axes and existing rotation centres in the design, cataloguing is done according to symmetry group theory.

- Simplification: the content of the fundamental parallelogram and cataloguing are sufficient to define the whole design, so that we will eliminate redundant information, without however dividing objects or groups.

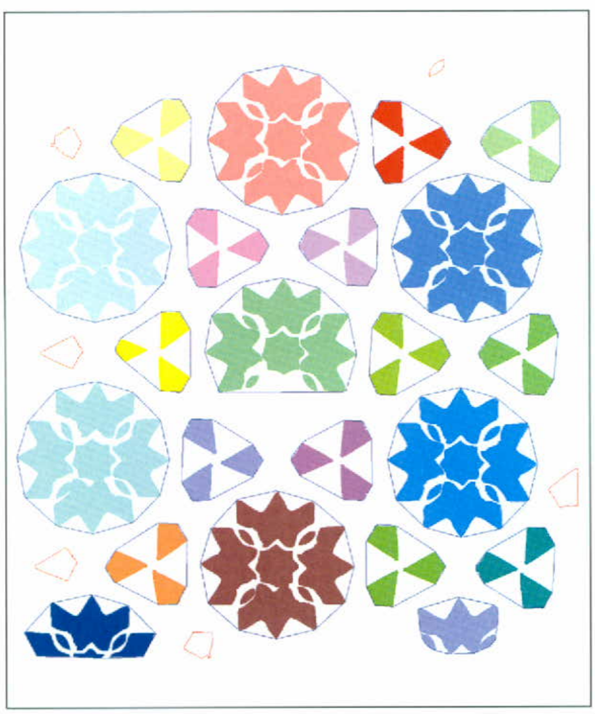


Figure 5: Groups found based on the image example (Group space).

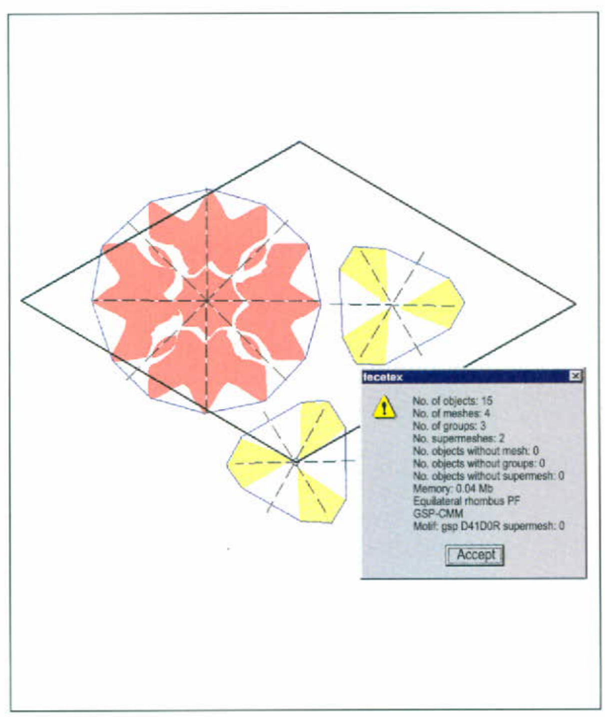


Figure 6: Fundamental parallelogram content and cataloguing of the example (Composition space).

3. APPLICATIONS TO THE DESIGN AND REDESIGN OF CERAMIC TILINGS.

The cataloguing results obtained not only enable identifying the tiling's type of compositional rhythm but also constitute an excellent source of information for ceramic tiling design and redesign. This information materialises in the PF content due to its following properties:

- Repeating it by translation according to the directions of its sides produces the whole design.
- It can be divided into a number of smaller regions, which by applying certain isometries generate it. These regions correspond to the tiles associated with a given isohedral tiling (IH)[5].
- Figure 7 shows the PF and the smallest regions into which it can be divided for each type of GSP. Each region is named according to the nomenclature that is used[5] for isohedral tilings (IH). In Figure 7 it can be observed that there are regions with an identical shape, which belong to different GSPs, such as the square region present in the GSPs: P1, PG, PM, CM, P2, PGG, PMG, PMM, CMM, P4, P4G and P4M. These regions with an identical shape differ from each other in the applied isometries.

To transfer these theoretical concepts to the field of ceramic tiling design, we propose considering regions with an identical shape, which belong to different GSPs, based on which the different compositional solutions are found according to the applied isometries. The study for the case of a square region can be observed in[26], and an amplification of this in[27].

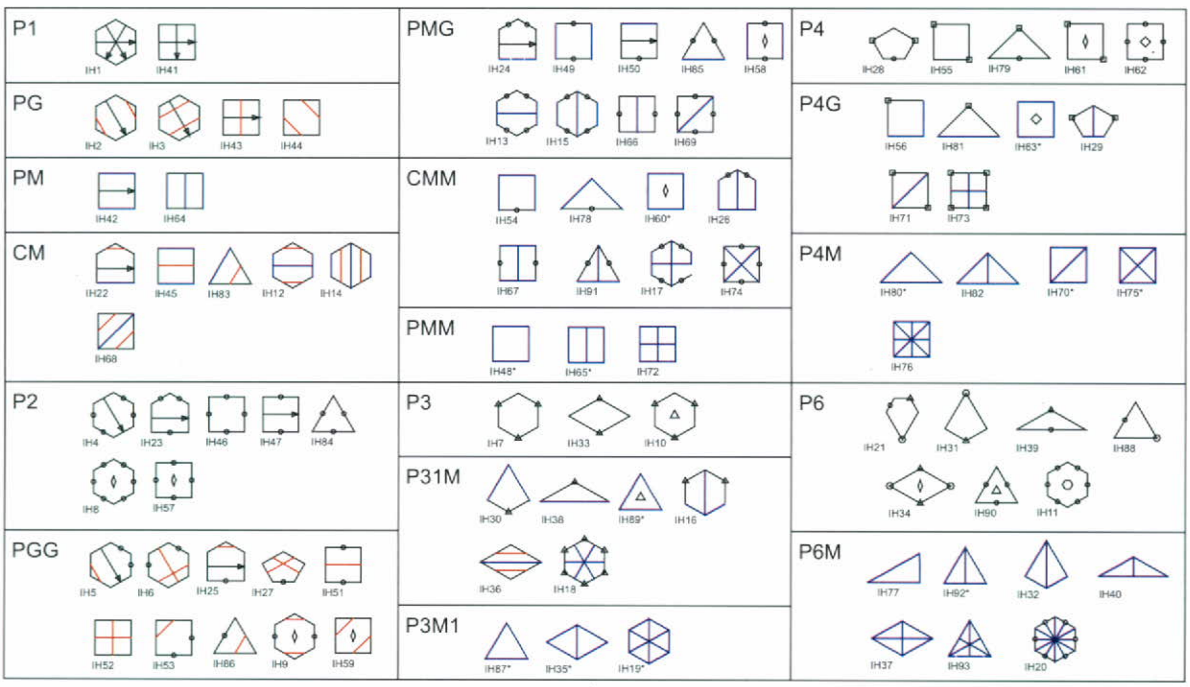


Figure 7: The PFs and regions of each GSP.

The outlined proposal allows two actions in relation to ceramic tiling design, depending on whether redesigns or new designs are sought.

Redesigns are achieved by working just with the PF found in cataloguing or with the smallest regions contained in the PF. Redesigns can be produced in the following ways:

1. Applying, to the PF or some of its regions, different isometries from those detected during cataloguing. With this action new compositions are found using pre-existing objects. Figure 12 gives an example of such a case. In this example, the PF and its content have been maintained of the result found in Figure 11, and the isometries applied to it have been modified.

2. Changing the content of the PF or of the region by relocating the elements that they contain. This can generate various compositions, depending on the application of certain isometries to the new PF or region. Figure 13 provides an example of such a case. In this example, the PF has been maintained of the result found in the Figure 11, altering its content by changing the colour of some of its components.

3. Modifying the figure of the PF or of the region, with or without variation of their contents. The redesigns are obtained on applying different isometries to the new PF or region associated with the chosen figure. Figure 14 depicts an example of such a case. In this example, the PF rhombus of the result obtained in Figure 10 has modified to a square PF, while the content has been maintained.

The new designs are made by fixing a type of PF or region and setting out the different compositions that can be produced depending on the isometries to be applied. They can be found in the following ways:

1. Composing a certain cluster of shapes (tiles or motifs) inside the PF or selected region. The new designs are obtained on applying different isometries to the PF or designed region. Figure 15 gives examples found of such a case, using a triangular region.

2. Modifying the polygonal figure of the PF or selected region in terms of the characteristic isometries of the type of GSP to be achieved. In this case it is feasible, after modifying the region figure, to compose a certain cluster of tiles inside it. Figure 16 sets out examples of such a case.

4. RESULTS

The digital images processed and catalogued using the developed tables come from three sources: photolibrary of the Instituto de Promoción Cerámica (IPC),of the Castellón County Council, historical catalogues digitised by this same Institute, and images obtained by scanning books containing collections of ceramic tilings[28].

Figures 8 and 9 present a list of the objects and groups of the example used in point 2, with their symmetry axes and the different orientations in which they appear. The plane symmetry group is CMM and the geometry of the fundamental parallelogram is an equilateral rhombus. Each of the different orientations of an object or group is depicted in a given colour.

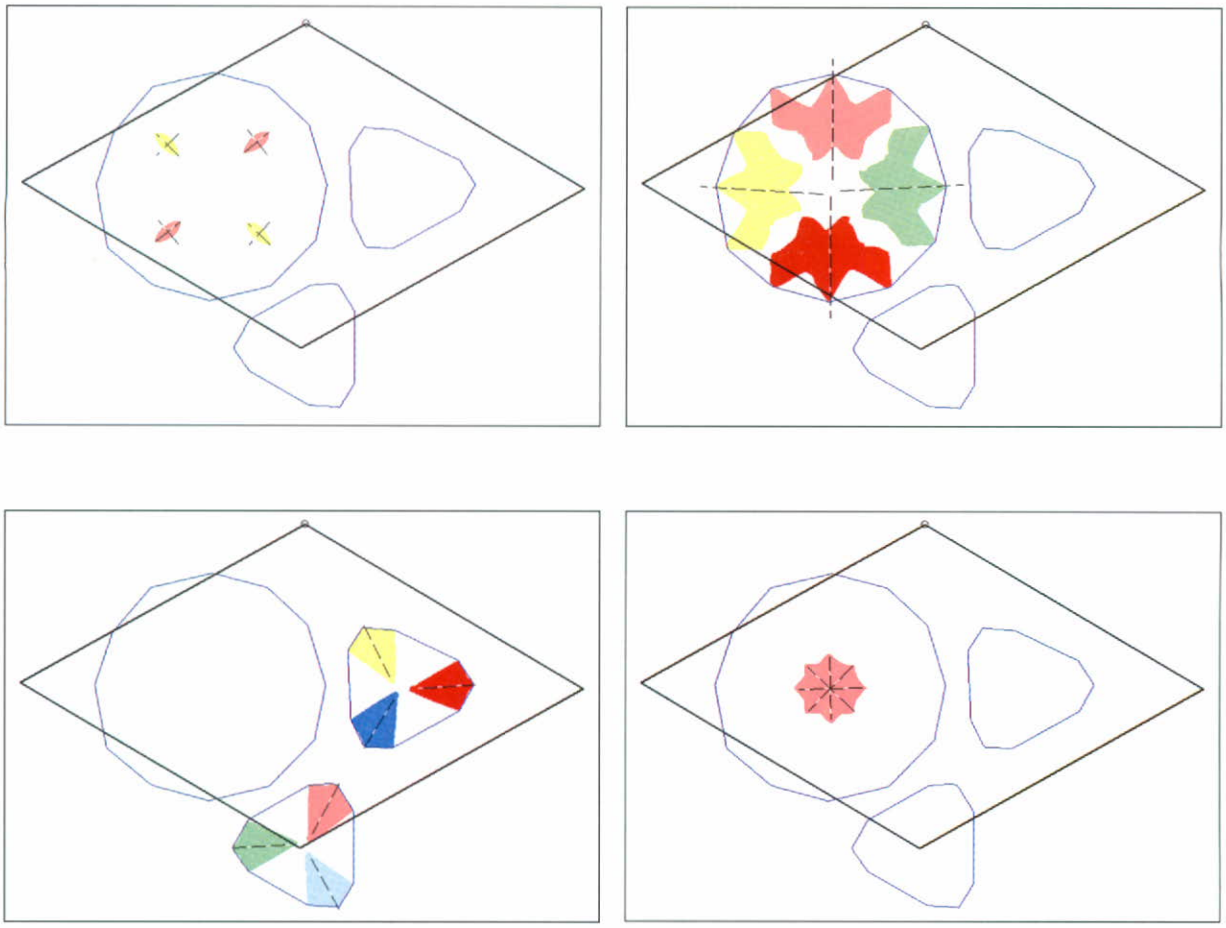


Figure 8: The four different types of objects with their symmetry axes (2, 1, 1 and 4 respectively).

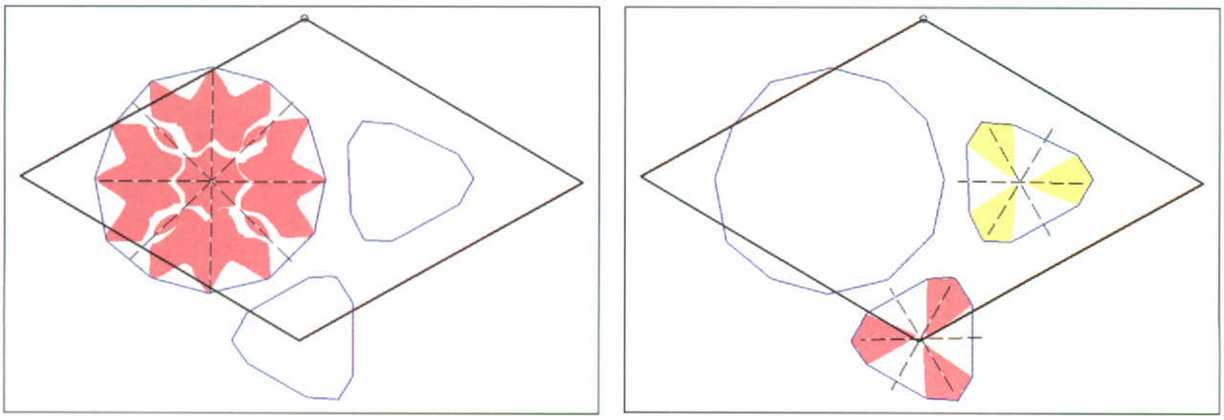


Figure 9: The two different types of groups with their symmetry axes (4 and 3 respectively).

Figures 10 and 11 show two original images with the resulting simplification and cataloguing of their analysis, and a later reconstruction of the tiling based on the simplified design.

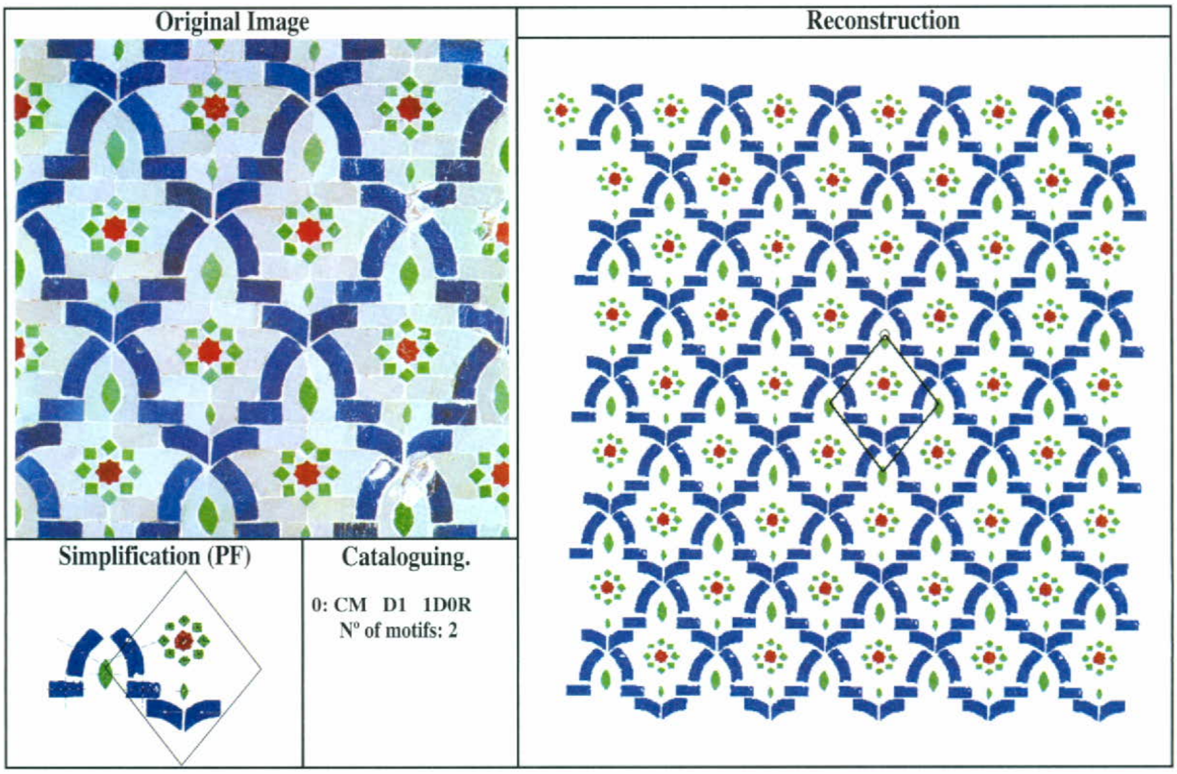


Figure 10: Cataloguing of a CM motif.

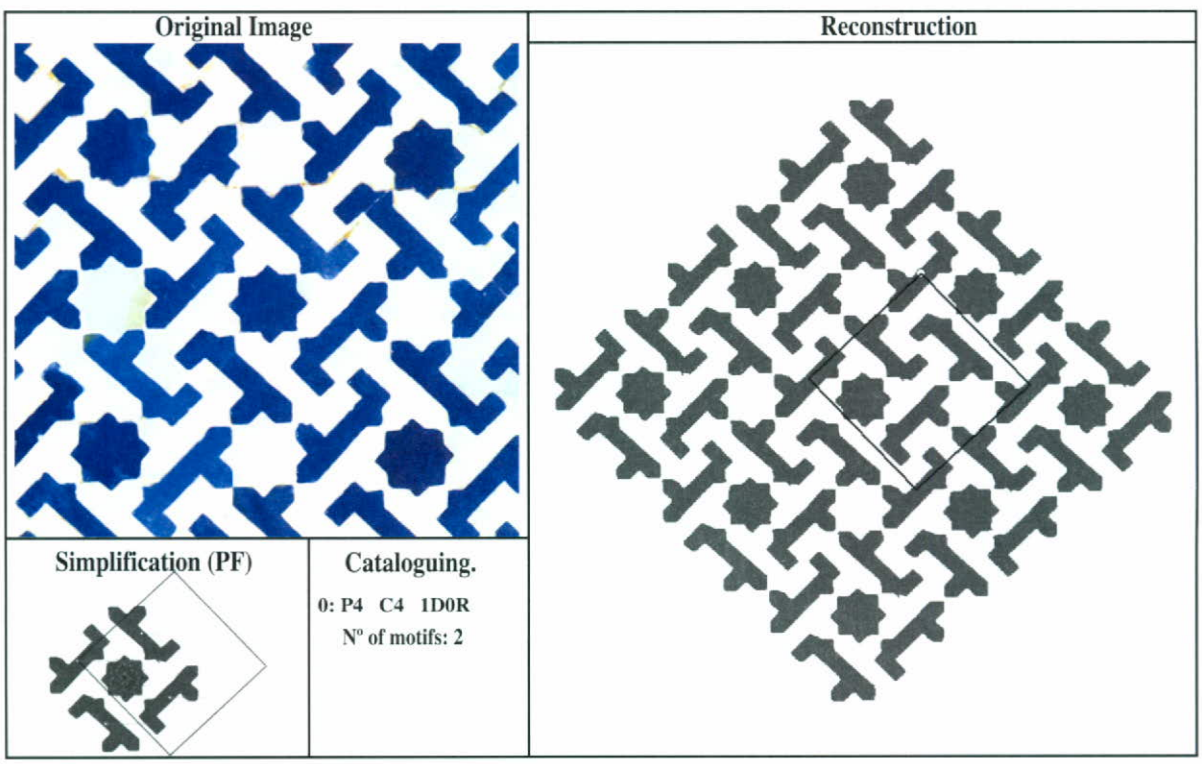


Figure 11: Cataloguing of a P4 motif.

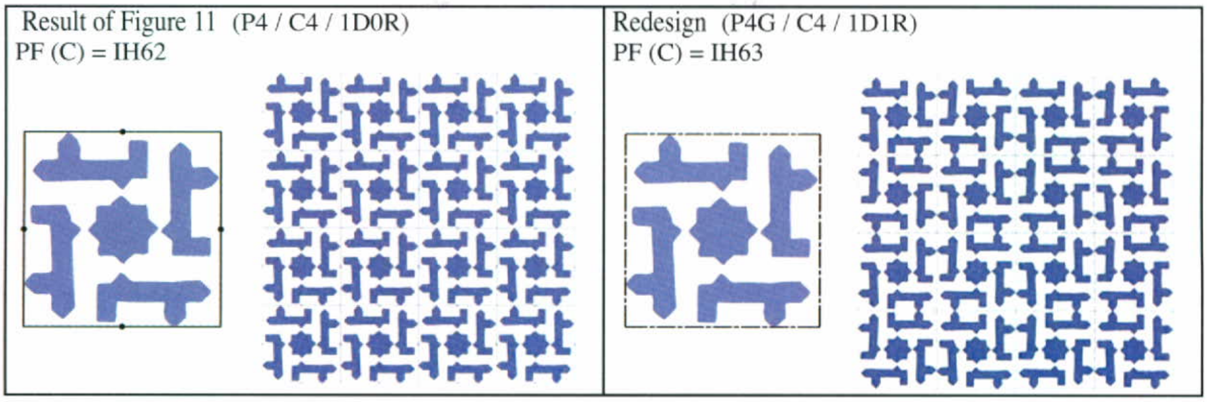


Figure 12: Redesigns maintaining PF.

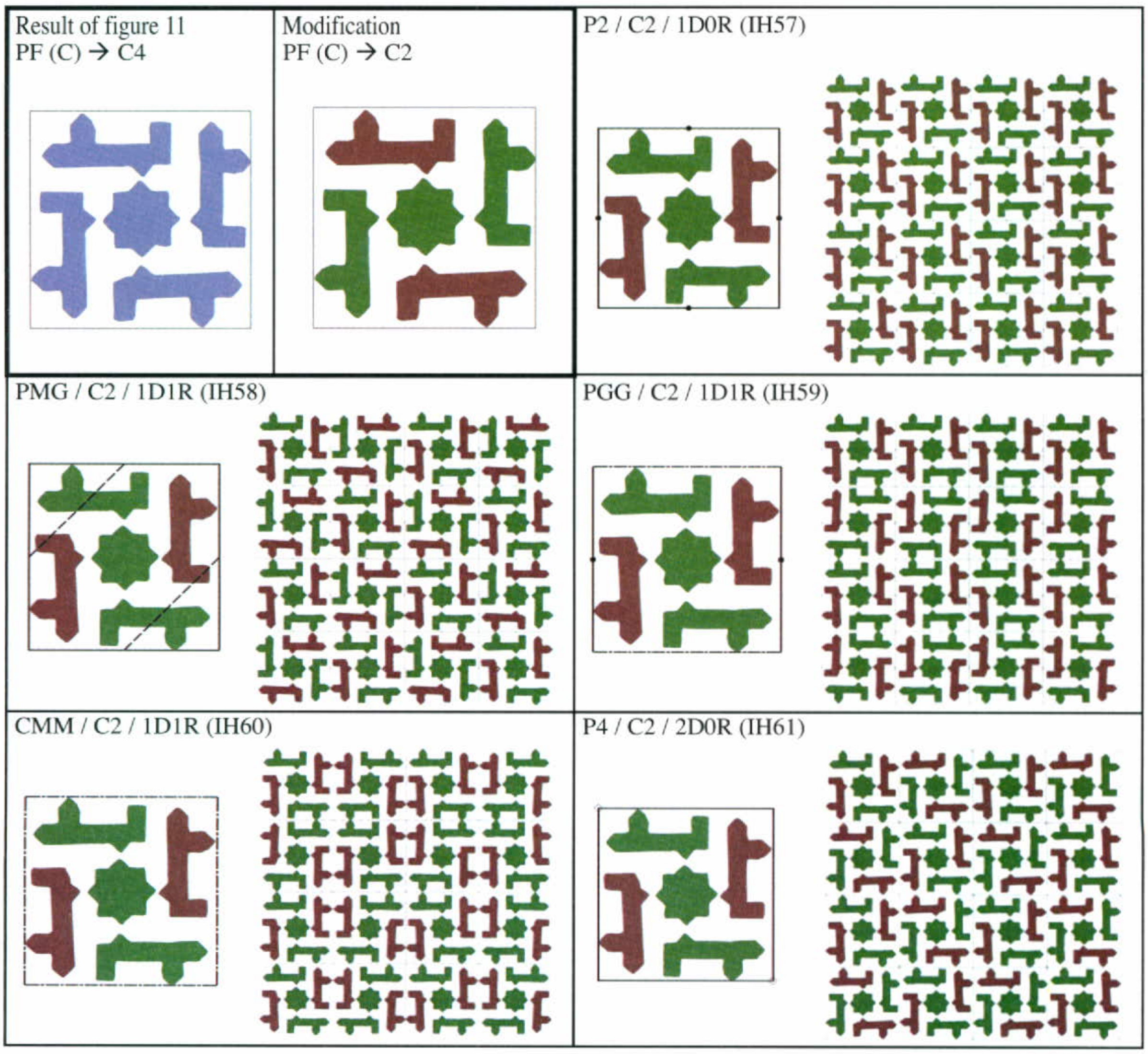


Figure 13: Redesigns modifying PF content

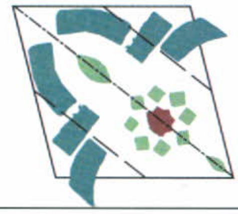
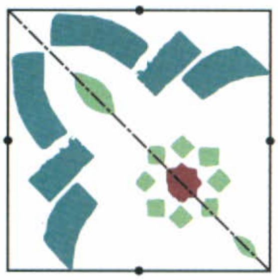
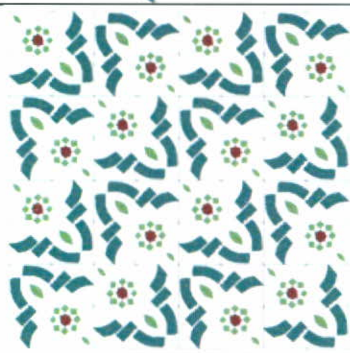
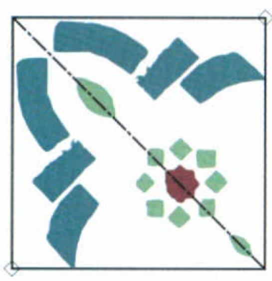
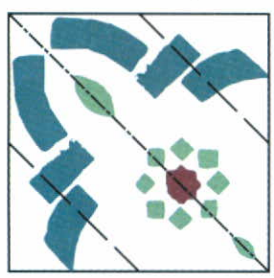
Result of figure 10 (CM / D1 / 1D0R) PF (RO) → IH78		
PF (C) → IH69	PMG / D1 / 2D0R 	
PF (C) → IH70	P4M / D1 / 4D0R 	
PF (C) → IH71	P4G / D1 / 4D0R 	
PF (C) → IH78	CM / D1 / 1D0R 	

Figure 14: Redesigns modifying PF and its content.

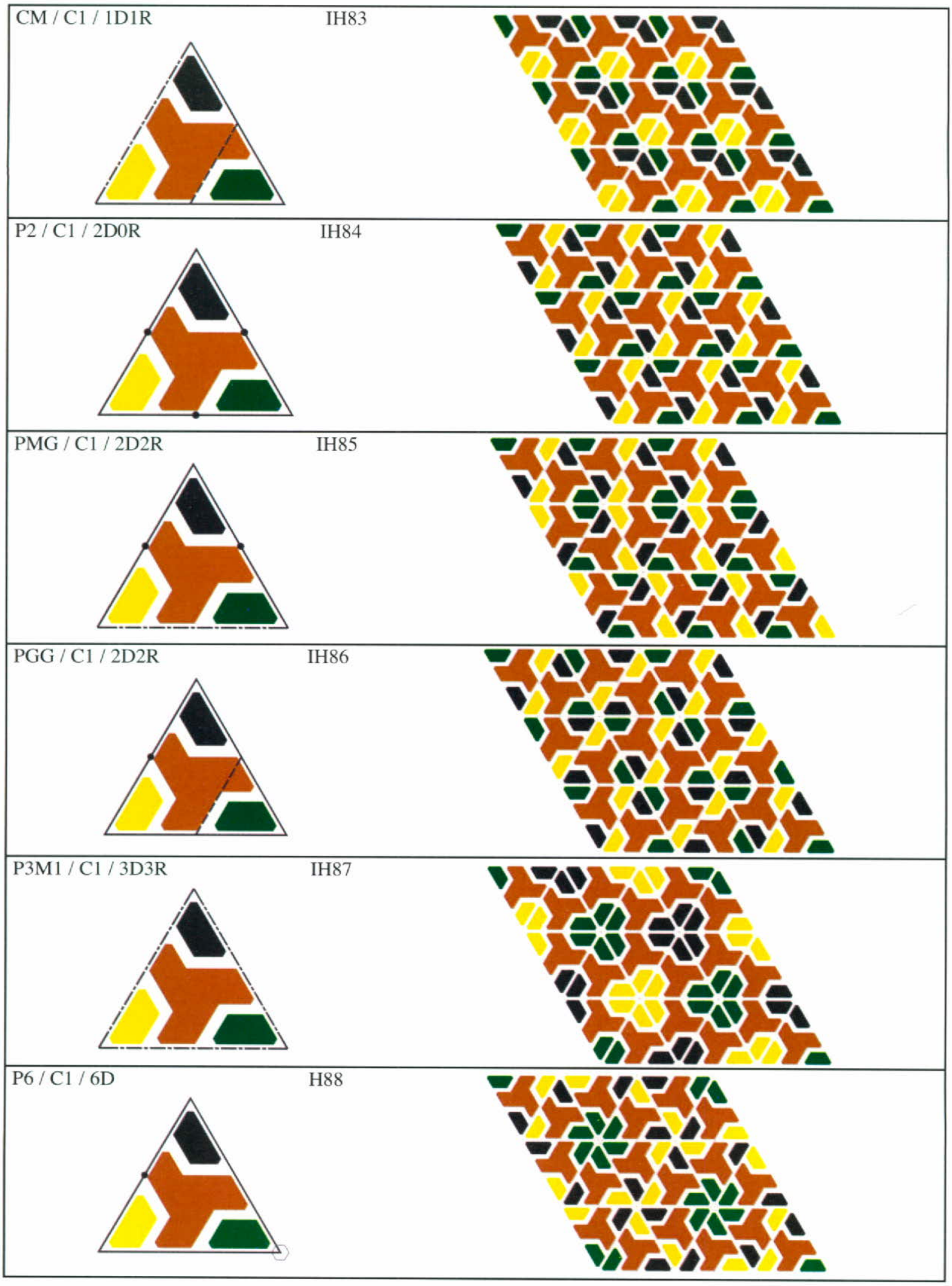


Figure 15: New designs grouping a set of tiles in a region.

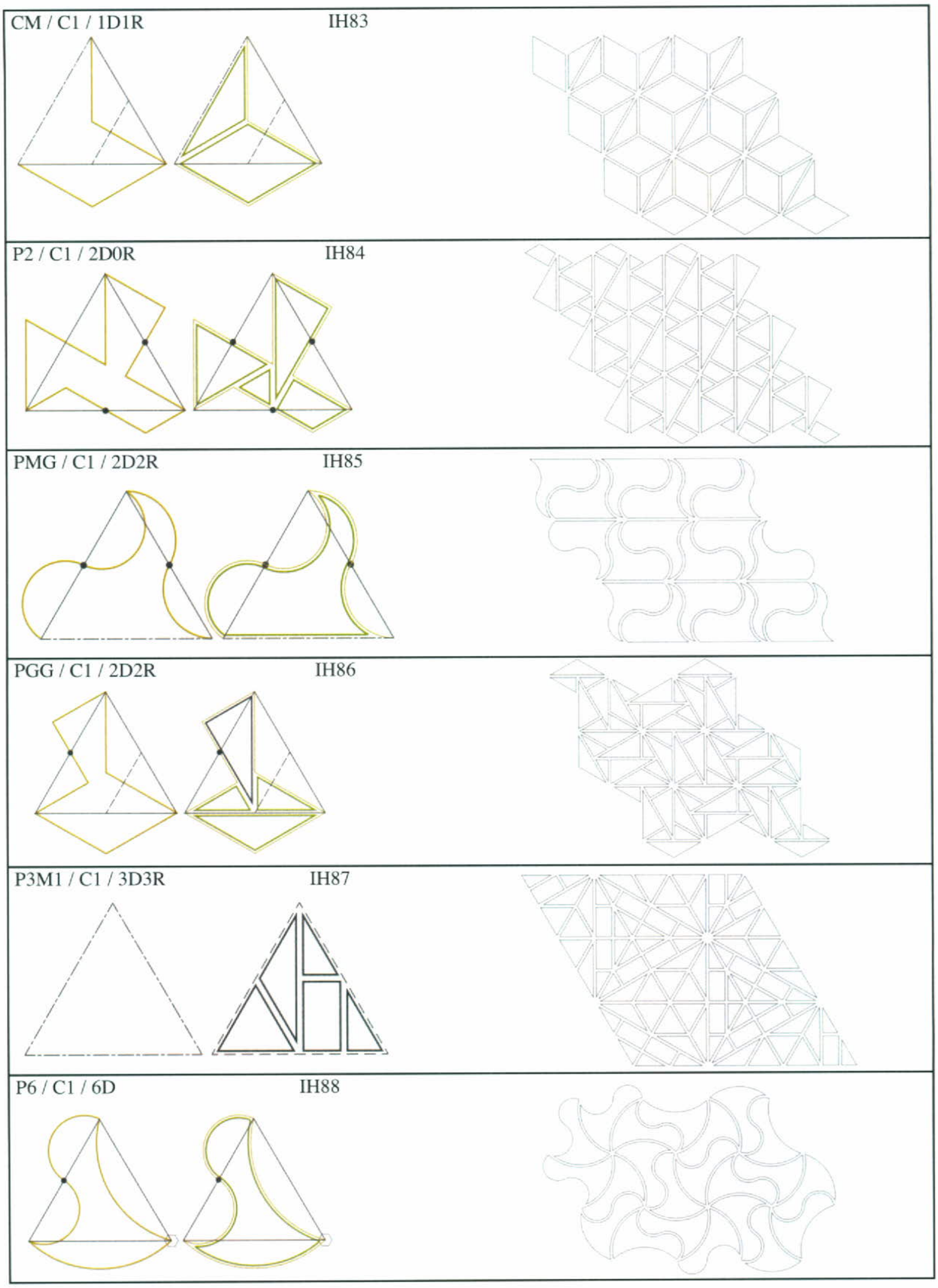


Figure 16: New designs modifying the figure of a region.

5. CONCLUSIONS AND AMPLIFICATIONS

In this paper a methodology has been presented for cataloguing ceramic tilings based on plane symmetry group theory. The methodology has been implemented in an analysis tool that includes operators for each of the different tasks it features.

GSP	IPC	BOOKS	CATALOGUES	TOTAL	%
P1	1			1	1.33
PG					
PM	1		1	2	2.77
CM		1		1	1.33
P2			1	1	1.33
PGG					
PMG					
PMM	1			1	1.33
CMM	1	1	2	4	5.53
P3		1		1	1.33
P31M					
P3M1					
P4	4		2	6	8.00
P4G			3	3	4.00
P4M	8	13	32	53	70.66
P6					
P6M		2		2	2.77
TOTAL	16	15	41	75	100%

Table 3: Results obtained.

The cataloguing tool has been applied to date to 75 ceramic tilings, yielding the results shown in Table 3. It is to be noted that 53 of these (70.66%) correspond to the P4M symmetry group. The reason for the prevalence of this group seems to be because of its own geometric structure, consisting of parallel reflections according to the sides and diagonals of a square. This structure corresponds to ceramic tilings produced using square tiles, which are obviously the most widely used owing to the ease of installation. The scarce presence of other groups such as PG, PGG, PMG, P31M, P3M1 and P6 evidence some reluctance to use certain compositions, mainly due to three facts:

1. They are more complex to install since the tiles need to be oriented as they are fixed, as in cases PG or PGG.
2. They cannot be made with square tiles, as in cases P31M, P3M1 or P6.
3. Their design requires certain mathematical knowledge, with which the designer is usually unfamiliar.

The tests carried out show that the tool achieves the objectives in a satisfactory way with most of the processed images. Only certain images, coming from photographs of old manufactured motifs (containing irregularities in the objects and colour differences), generally deteriorated (dirt and breakages), cannot be analysed correctly. The borders of the pieces can also be problematic. Nevertheless, the operators that incorporate reconstruction (comparing and uniforming groups, and reconstruction by the fundamental parallelogram) enable solving some of the previously mentioned problems, whenever these are not too decisive

The tests performed show that on an image level, the images must have sufficient resolution. That is to say, an object needs to be represented by a large number of pixels, so that the size of the pixels can be disregarded and the existence or absence of certain pixels in different representations of the same object is not decisive. It is also necessary for the image to contain enough repetitions to enable extracting the fundamental parallelogram. These two requirements: greater resolution and larger extension require bigger images, needing larger memories and greater processing speed. Unprocessable image sizes can occur, making it necessary to adopt a compromise solution, for example, by not taking into account the smallest objects and only working with sufficiently large ones.

As amplifications of the analysis tool, it is sought to obtain the minimum region (part that can regenerate the design based on shifts, rotations and reflections), as well as to use the cataloguing information (axes of symmetry of objects and groups and plane symmetry group) to reconstruct the image.

Further, different applications of the results of the cataloguing have been set out in design and redesign tasks in ceramic floor and wall tilings. The importance of these applications in the design areas of the companies in this sector allows considering implementing new design methodologies that foster designer creativity. These applications form the core of the research work currently being undertaken by the authors.

Finally, it should be noted that the tools and applications described, using information systems such as SIMOD which facilitate retrieval of a great volume of graphic information on ceramic tilings, facilitate the integration of all the activities relating to two-dimensional decorative motifs in a single computer-aided design environment.

REFERENCES

[1] CARRETERO, M., VALIENTE, J.M., ALBERT, F., GOMIS, J.M. "Diseño de un sistema de información gráfica para tareas de rediseño industrial", Proceedings of the XIII Congreso Internacional de Ingeniería Gráfica (Ingegraf), Badajoz, June 2001. pp 33.

[2] ALSINA, C.; TRILLAS, E. "Lecciones De Álgebra Y Geometría", Gustavo Gili, Barcelona, 1992.

[3] BUDDEN, F.J. "The Fascination Of Groups", Cambridge University Press, 1972.

[4] COXETER, H.S.M. "Fundamentos De Geometría, Limusa, Mexico, 1984.

[5] GRÜNBAUM, B., SHEPHARD, G.C. "Tilings and Patterns", W. H. Freeman, New York, 1987.

[6] MARTINN, G.E. "Transformation Geometry. An Introduction To Symmetry", Springer-Verlag, New York, 1982.

[7] SCHATTSCHNEIDER, D. "The Plane Symmetry Groups: Their Recognition And Notation", The American Mathematical Monthly, 1978, 85, 439-450.

[8] SHUBNIKOV, A.V.; KOPTSIK, V.A. "Symmetry In Science And Art", Plenum Press, New York, 1974.

[9] WELY, H. "Symmetry", Princeton University Press, New Jersey, 1989..

[10] YALE, P.B. "Geometry And Symmetry", Holen-Day, San Francisco, 1968.

[11] PÉREZ GUILLÉN, I.V. "Cerámica Arquitectónica Valenciana. Los Azulejos De Serie. Siglos XVI-XVIII", Vol. 1-2, Institut de Promoció Cerámica de Castello, 1996, ISBN 84 4821 35 72.

[12] PÉREZ GUILLÉN, I.V. " Azulejos Valencianos De Serie. El Siglo XIX", Vol. 1-2, Institut de Promoció Cerámica de Castello, 2001.

[13] VALIENTE, J.M., AGUSTÍ, M., GOMIS, J.M., CARRETERO, M. "Estudio de técnicas de segmentación de imágenes en color aplicadas al diseño textil", Proceedings of the XIII Congreso Internacional de Ingeniería Gráfica (Ingegraf), Badajoz, June 2001. pp 34.

[14] COMANICIU, D. ; MEER, P. "Distribution Free Decomposition of Multivariate Data" In Pattern Analysis andApplications, Vol. 2, pp 22-30, 1999.

[15] COMANICIU, D. ; MEER, P. "Robust analysis of feature spaces: Color image segmentation". In Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition., IEEE, Los Alamitos, CA, USA, 1997. pp. 750-755.

[16] CHENG, Y. "Mean Shift, mode seeking, and Clustering", IEEE Trans. On Pattern Anal. Machine Intelligence. PAMI Vol. 17, 790-799, 1995.

[17] ALBERT, F., GOMIS, J.M., VALOR, M., VALIENTE, J.M., CARRETERO, M. "Análisis estructural de motivos decorativos en diseño textil", Proceedings of the XIII Congreso Internacional de Ingeniería Gráfica (Ingegraf), Badajoz, June 2001. pp 35.

[18] GOMIS J. M.; ALCAÑIZ M.; DÍAZ C.; HERRERAS M. A.; Landete M..; "La Vectorización y la Gestión de Documentos en los Sistemas CAD". Proceedings of the X Congreso Internacional de Ingeniería Gráfica (Ingegraf), Malaga, June 1998. Vol I, pp 565-582.

[19] GONZÁLEZ R. C., WINTZ P. "Digital Image Processing", Addison-Wesley, 1987.

[20] BARTELS R. H., BEATTY J. C., BARSKY B. A. "An Introduction to Splines for Use in Computer Graphics & Geometric Modeling", Los Altos (USA): Morgan Kaufmann, 1987.

[21] ALBERT, F., GOMIS, J.M., CARRETERO, M., VALIENTE, J.M. "Object comparison in the structural analysis of textile design patterns", Proceedings of the XII ADM International Conference on Design Tools and Methods in Industrial Engineering, Rimini (Italy), September 2001. pp 77.

[22] ARNHEIM, R. "Arte y Percepción visual (Psicología de la visión creadora)", Trillas, 1972.

[23] COHEN, J. "Sensación y percepción visuales", Trillas, 1979.

[24] GUILLAUME, P. "Psicología de la forma", Psique, 1971.

[25] KÖHLER, W., KOFFKA, K., SANDER, F. "Psicología de la forma", Paidós, 1969.

[26] GOMIS, J.M.; VALOR, M. "Application of the Theory of Symmetry Groups to the Cataloguing of Square Ceramic Tilings ", IV World Congress on Ceramic Tile Quality, Cámara Oficial de Comercio, Industria y Navegación de Castellón, 1996.

[27] VALOR, M., GOMIS, J.M., "Paralelogramos Fundamentales En La Cubrición Del Plano Mediante Formatos Rectangulares". VIII Congreso Internacional de Ingeniería Gráfica. Publications of University of Jaen, 1996, 1, 145-160.

[28] PACCARD, A. "Le Maroc Et L´Artisanat Traditionel Islamique Dans L´Architecture", Éditions Atelier 74, Bonlieu, 1983, ISBN 2-86.486-009-0.